

Addition, subtraction and scalar multiplication of matrices

sigma-matrices3-2009-1

This leaflet will look at the condition necessary to be able to add or subtract two matrices, and when this condition is satisfied, how to do this. It will also cover how to multiply a matrix by a number.

Compatible matrices

Two matrices are said to be compatible when they have the same size, that is, the same number of rows and the same number of columns. When two matrices are compatible they can be added (or subtracted).

Here are four matrices.

$$A = \begin{pmatrix} 4 & 3 \\ 0 & -1 \end{pmatrix} \quad B = \begin{pmatrix} 5 & -2 \\ -1 & 3 \\ 1 & 0 \end{pmatrix} \quad C = \begin{pmatrix} 3 & 0 & -3 \\ 0 & 2 & 1 \end{pmatrix} \quad D = \begin{pmatrix} 2 & -2 \\ 0 & 1 \\ 4 & -1 \end{pmatrix}$$

First consider the size of each of these matrices:

A is a 2×2 matrix, B is 3×2 , C is 2×3 and D is 3×2 .

From these four matrices only B and D are compatible. This means we can calculate $B + D$, $B - D$, $D - B$, but we cannot add or subtract any other pair of these matrices.

The matrix $C^T = \begin{pmatrix} 3 & 0 \\ 0 & 2 \\ -3 & 1 \end{pmatrix}$ is a 3×2 matrix, so we could add or subtract C^T to/from B and D .

Adding and subtracting matrices

When two matrices are compatible we add (or subtract) them by adding (or subtracting) the elements in corresponding positions.

We have seen that B and D have the same size; they are both 3×2 matrices. This means we can add them by adding elements in the corresponding (that is, the same) positions:

$$B + D = \begin{pmatrix} 5 & -2 \\ -1 & 3 \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} 2 & -2 \\ 0 & 1 \\ 4 & -1 \end{pmatrix} = \begin{pmatrix} 5+2 & -2+(-2) \\ -1+0 & 3+1 \\ 1+4 & 0+(-1) \end{pmatrix} = \begin{pmatrix} 7 & -4 \\ -1 & 4 \\ 5 & -1 \end{pmatrix}$$

Note that the answer we get is another matrix of the same size as the ones we started with.

Similarly, we can find $B - D$ by subtracting elements in the corresponding positions.

$$B - D = \begin{pmatrix} 5 & -2 \\ -1 & 3 \\ 1 & 0 \end{pmatrix} - \begin{pmatrix} 2 & -2 \\ 0 & 1 \\ 4 & -1 \end{pmatrix} = \begin{pmatrix} 5-2 & -2-(-2) \\ -1-0 & 3-1 \\ 1-4 & 0-(-1) \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ -1 & 2 \\ -3 & 1 \end{pmatrix}$$

Since any matrix is the same size as itself we can always add a matrix to itself. If we do this for matrix A we find

$$A + A = \begin{pmatrix} 4 & 3 \\ 0 & -1 \end{pmatrix} + \begin{pmatrix} 4 & 3 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 4+4 & 3+3 \\ 0+0 & -1+(-1) \end{pmatrix} = \begin{pmatrix} 8 & 6 \\ 0 & -2 \end{pmatrix}$$

Another way to write $A + A$ is $2A$. Therefore we have

$$2A = \begin{pmatrix} 8 & 6 \\ 0 & -2 \end{pmatrix}$$

We see that the elements of $2A$ are each twice the elements of A . This illustrates how to multiply a matrix by a number, and leads us to the topic of **scalar multiplication**:

Scalar multiplication

When working with matrices there are two kinds of multiplication: scalar multiplication and matrix multiplication. Scalar multiplication is where a matrix is multiplied by a single number. Matrix multiplication is where a matrix is multiplied by another matrix. - this is covered in a later leaflet.

To multiply a matrix by a scalar (that is, a single number), we simply multiply each element in the matrix by this number.

Using the matrices above we have the following:

$$5B = \begin{pmatrix} 5 \times 5 & 5 \times -2 \\ 5 \times -1 & 5 \times 3 \\ 5 \times 1 & 5 \times 0 \end{pmatrix} = \begin{pmatrix} 25 & -10 \\ -5 & 15 \\ 5 & 0 \end{pmatrix}$$

$$-3A = \begin{pmatrix} -3 \times 4 & -3 \times 3 \\ -3 \times 0 & -3 \times -1 \end{pmatrix} = \begin{pmatrix} -12 & -9 \\ 0 & 3 \end{pmatrix}$$

$$\frac{1}{2}D = \begin{pmatrix} \frac{1}{2} \times 2 & \frac{1}{2} \times (-2) \\ \frac{1}{2} \times 0 & \frac{1}{2} \times 1 \\ \frac{1}{2} \times 4 & \frac{1}{2} \times (-1) \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 0 & \frac{1}{2} \\ 2 & -\frac{1}{2} \end{pmatrix}$$

$$4C = \begin{pmatrix} 4 \times 3 & 4 \times 0 & 4 \times -3 \\ 4 \times 0 & 4 \times 2 & 4 \times 1 \end{pmatrix} = \begin{pmatrix} 12 & 0 & -12 \\ 0 & 8 & 4 \end{pmatrix}$$

Further leaflets in this series will explain other operations we can carry out with matrices.

Note that a video tutorial covering the content of this leaflet is available from **sigma**.