The slope-intercept form

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Introduction

One form of the equation of a straight line is called the **slope-intercept** form because it contains information about these two properties.

The equation of a straight line

Any equation of the form

$$y = mx + c$$

where m and c are fixed numbers, (i.e. constants), has a graph which is a straight line.

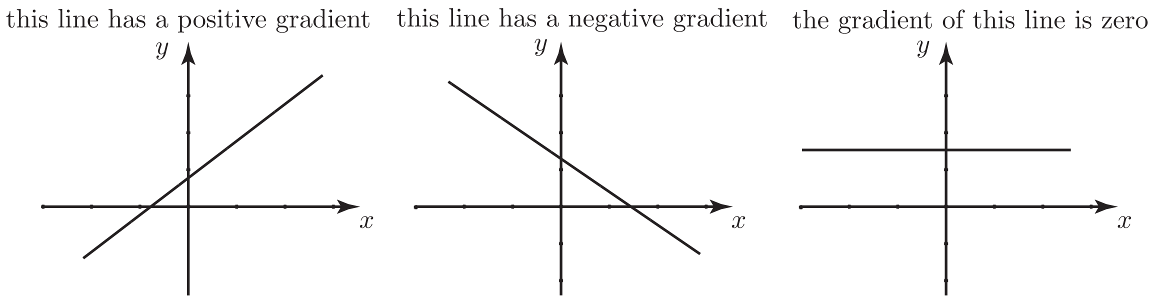
For example,

$$y = 3x + 5, \qquad y = \frac{2}{3}x + 8 \qquad \text{and} \qquad y = -3x - 7$$

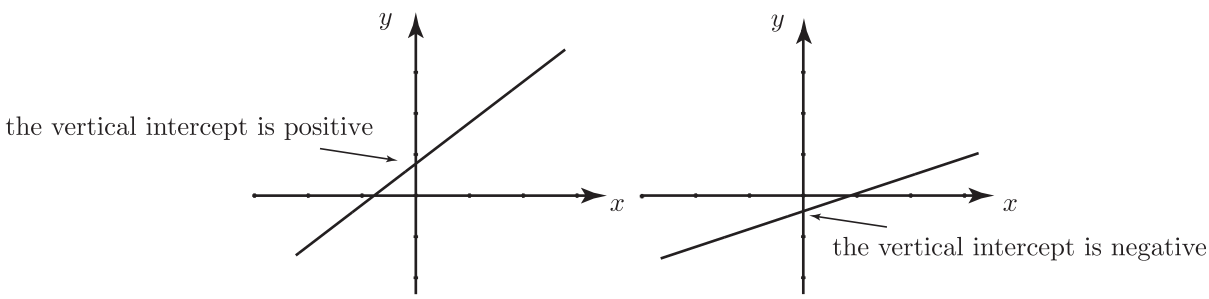
all have graphs which are straight lines.

The slope and intercept of a straight line

In the equation $y = mx + c$ the value of m is called the **slope**, (or gradient), of the line. It can be positive, negative or zero. Lines with a positive gradient slope upwards, from left to right. Lines with a negative gradient slope downwards from left to right. Lines with a zero gradient are horizontal.



The value of c is called the **vertical intercept** of the line. It is the value of y when $x = 0$. When drawing a line, c gives the position where the line cuts the vertical axis.



Example

Determine the gradient and vertical intercept of each line.

a) $y = 12x - 6$, b) $y = 5 - 2x$, c) $4x - y + 13 = 0$, d) $y = 8$, e) $y = 4x$.

Solution

a) Comparing $y = 12x - 6$ with $y = mx + c$ we see that $m = 12$, so the gradient of the line is 12. The fact that this is positive means that the line slopes upwards as we move from left to right. The vertical intercept is -6. This line cuts the vertical axis below the horizontal axis.

b) Comparing $y = 5 - 2x$ with $y = mx + c$ we see that $m = -2$, so the gradient is -2. The line slopes downwards as we move from left to right. The vertical intercept is 5.

c) We write $4x - y + 13 = 0$ in standard form as $y = 4x + 13$ and note that $m = 4$, $c = 13$.

d) Comparing $y = 8$ with $y = mx + c$ we see that $m = 0$ and $c = 8$. This line is horizontal.

e) Comparing $y = 4x$ with $y = mx + c$ we see that $m = 4$ and $c = 0$.

Exercises

1. State the gradient and intercept of each of the following lines.

a) $y = 5x + 6$, b) $y = 3x - 11$, c) $y = -2x + 7$, d) $y = 9$, e) $y = 7 - x$

Answers

1. a) gradient 5, intercept 6 b) 3,-11, c) -2,7, d) 0,9, e) -1, 7.

More about the gradient

The gradient measures the steepness of the line. A large positive value of m means the graph increases steeply as you move from the left to the right. A small, but positive value of m means the graph increases, but not very steeply. Similarly, a large negative value of m means that the graph drops steeply as you move from left to right. A small negative value means the graph decreases, but not very steeply.

In fact we can say more. The value of m tells us the amount by which y increases (or decreases) if x increases by one unit.

For example, for the line $y = 5x + 13$, the value of y increases by 5 units every time x increases by 1 unit.

In the line $y = -3x + 7$ the value of y decreases by 3 units every time x increases by 1 unit. You should sketch these graphs to convince yourself of this behaviour.

Exercises

1. If $P = 4Q + 9$, by what amount will P increase if Q increases by 1 unit ?

2. If $P = 11 - 3Q$, by what amount will P decrease if Q increases by 1 unit ?

3. If $P = 19$, by what amount will P increase if Q increases by 1 unit ?

Answers

1. 4. 2. 3.

3. It will not. The value of P is constant, that is fixed at 19. It does not depend upon Q.