Linear relationships

Introduction

In many business applications two quantities are related linearly. This means that a graph of their relationship takes the form of a straight line. This leaflet discusses one form of the mathematical equation which describes linear relationships.

Linear equations

If x and y are two variables, and a, b and c are constants, then an equation relating x and y which takes the form

$$ax + by = c$$

is said to be a **linear equation**.

The following are linear equations because they are of this form:

$$3x + 2y = 7, \qquad 4x - 8y = 2, \qquad -2x + y = 9, \qquad \frac{2}{3}x + y = 3$$

Examine each example in turn to find the values of the constants a, b and c. Note that these constants can be positive or negative, and can be fractions. The values of a and b will never both be zero. When graphs of these equations are plotted they will be straight lines. This is the reason for the term *linear* equation. We will see how to do this shortly.

The following are not linear equations:

$$2x + \frac{1}{y} = 7, \qquad 7x^2 + 3y = 0, \qquad \sqrt{x} + 8y = 9$$

The first is not linear because of the term $\frac{1}{y}$. The second is not linear because of the term x^2. Finally, the third example is not linear because of the term $\sqrt{x}$. When graphs of these equations are plotted they will be curves, rather than straight lines.

The graph of a linear equation

Any straight line graph can be drawn by plotting just two points which satisfy the linear equation and then joining them with a straight line.

Consider the examples which follow:

Example

Find two points which satisfy the equation $3x + 4y = 24$ and hence plot its graph.
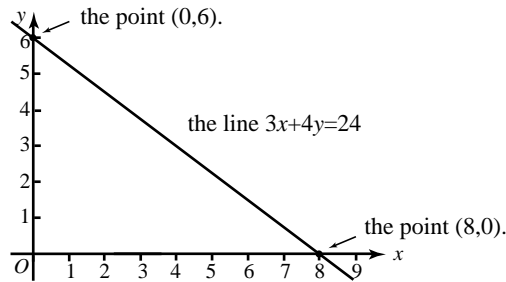
Solution

Two points are easily found as follows:

First set $x = 0$. Then the equation becomes $4y = 24$, that is $y = \frac{24}{4} = 6$. This means that when $x = 0$, the value of y is 6. So the point with coordinates $(0, 6)$ lies on the line.

Next set $y = 0$. Then the equation becomes $3x = 24$, that is $x = \frac{24}{3} = 8$. This means that when $y = 0$, the value of x is 8. So the point with coordinates $(8, 0)$ lies on the line.

These two points are plotted, and shown as •, and then joined together to form the straight line graph as shown below. The line slopes downwards as we move from left to right.


Example

Find two points which satisfy the equation $2y - 3x = 3$ and hence plot its graph.

Solution

First set $x = 0$. Then the equation becomes $2y = 3$, that is $y = \frac{3}{2}$. This means that when $x = 0$, the value of y is $\frac{3}{2}$. So the point with coordinates $(0, \frac{3}{2})$ lies on the line.

Next set $y = 0$. Then the equation becomes $-3x = 3$, that is $x = \frac{3}{-3} = -1$. This means that when $y = 0$, the value of x is -1. So the point with coordinates $(-1, 0)$ lies on the line.

These two points are plotted, and shown as •, and then joined together to form the straight line graph as shown below. This time the line slopes upwards as we move from left to right.
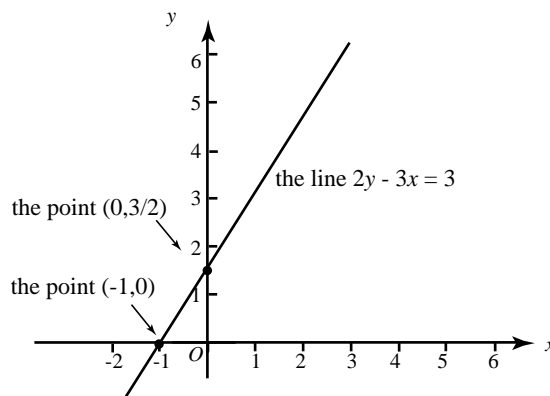
Exercises

Sketch the following straight line graphs: (a) $4y + 2x = 12$, (b) $5x - 15y = 25$.