Good Practice on Inclusive Curricula in the Mathematical Sciences

Mathematical Sciences HE Curriculum Innovation Project

Edited by Emma Cliffe and Peter Rowlett

The Higher Education Academy Maths, Stats & OR Network

National HE STEM Programme

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Contents

Introduction: good practice on inclusive curricula in the mathematical sciences

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Background

Inclusive curriculum practice refers to "the process of developing, designing and refining programmes of study to minimise the barriers that students may face in accessing the curriculum" (Higher Education Academy [1]). In addition to reacting to the needs of individual students through reasonable adjustments, potential barriers should be anticipated and resolved in a proactive manner. Both aspects are part of the legal requirement [2]. A focus on the core requirements of the course allows potential barriers to be identified, so that course delivery can be redesigned to remove or reduce those barriers.

Courses with substantial mathematical content pose specific accessibility challenges beyond those usually considered in generic inclusive curricula good practice advice (see for example the series of articles in MSOR Connections [3]). The Mathematics, Statistics and OR (MSOR) Access Working and Interest Group (AccessMSOR WG) [3], of which the author is Chair, brings together expertise and interest in issues surrounding the support of disabled students in MSOR subjects. An AccessMSOR WG workshop on inclusive curricula in MSOR took place with support from the National HE STEM Programme in February 2011. Following this, group members and workshop attendees were invited to submit case studies or reports relevant to the theme.

The resultant guide, 'Good Practice on Inclusive Curricula in the Mathematical Sciences', seeks to complement and extend, rather than replace, generic good practice advice. In order to produce a practical document, in addition to raising awareness of key questions, contributors were asked to provide a context, identify potential barriers and give clear recommendations.

Potential barriers in MSOR

MSOR subjects are cumulative in nature so concepts may take time to be fully assimilated and this has an impact on teaching and assessment design. The Subject Benchmark Statement [4] notes that seeing extended arguments developed in 'real-time' is of benefit and this underlies the continuing dominance in some contexts of traditional board-based lectures. These may be supported by tutorials, problem classes and seminars which may also be board-based.

Many of the contributions to the guide highlight the need for full notes in specific formats to be provided prior to classes. For some, for example a student reading in Braille or large print, these notes may act as an alternative to the board. For others, the provision of notes alters the main activity from copying precisely, maintaining place in the text or retaining definitions in working memory to the desired engagement with concepts and logical arguments. Without this provision, the 'real-time' development of the argument cannot be followed. However, the very use of the board to facilitate this 'real-time' development may mean that no notes exist which can be converted for these students. This conundrum does not arise in the majority of subjects where extensive board use has all but disappeared.

This issue forms part of a wider challenge for the MSOR community, that documents and websites which contain mathematical content may themselves be a barrier. The Subject Benchmark Statement says that assessment should not be restricted to knowledge and understanding alone but also to the ability to communicate effectively in the context of MSOR. Reading and assimilating ideas typically expressed in two dimensional symbolic notation interspersed with text and diagrams is one specialist element of MSOR communication, along with developing effective written and oral communication of MSOR material with both peers and staff. This might include, for instance, use of LaTeX, a format used for the typesetting of scientific documents. However, no single format has yet emerged which can be read or transformed to be read effectively by all and this barrier is a recurring theme throughout the contributions to the guide.

Differing perspectives

A student may draw on support from needs assessors, assistive technology trainers, disability advisers, specialist mentors and study skills tutors, librarians, careers advisers, study support, examinations support and document conversion staff. E-learning specialists and computing services may be responsible for ensuring access to the virtual learning environment, computer systems and software. Most of these support professionals will not have substantial experience of mathematical subjects and, not unreasonably, may assume that generic approaches to access and inclusive design remain valid.

For example, it may be incorrectly assumed: that all electronic resources are accessible; that Braille, large or alternative print and speech formats can be produced automatically; that staff will typically provide documents in editable electronic formats; that standard optical character recognition and voice recognition software works; that students will know how to use software such as literacy support and mindmapping programs when faced with a proof or partial differentiation question; and, that standard study support tutorials will be effective.

Meanwhile, lecturers and tutors in mathematics, unlike their counterparts at a specialist school, are likely to have only limited knowledge in the domains of the support professionals listed above. Not unreasonably, they may assume that the student has been provided with assistive technology, training, human support and advice appropriate to the specialist nature of their studies and the ways in which MSOR content is communicated.

Understanding of the nature of mathematics, how it is communicated, taught and assessed, rests with the subject department. The contributions to the guide evidence the value of support professionals developing some understanding of the specialist nature of mathematics and of departments developing their technical and pedagogic offering in awareness of access challenges.

This leads to the recommendation that students, MSOR staff and support professionals should collaborate to identify MSOR specific barriers, find effective solutions and ultimately design inclusive curriculum delivery for the future.

Good practice guide

The good practice guide necessarily draws on the particular knowledge and interests of its contributors and cannot claim to provide a comprehensive picture. Nevertheless, with contributions from different stakeholders – academic staff, professional support staff, disability researchers and students – the guide aims to be a step towards the goal of working together to develop inclusive curricula.

The guide concludes with a collection of references to resources, sources of further information and key papers with short annotations. This list is provided to assist departments seeking MSOR specialist information to discover resources more effectively.

Common threads that run through the contributions indicate common challenges for inclusive practice in MSOR. Contributions explore technical and pedagogic barriers and the way these may be formed by the modes in which mathematics is communicated. The contributions provide strong evidence of the need for collaboration between the MSOR community and the support professionals in dissolving these barriers and moving together towards the goal of inclusive curricula.

References

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Experiences of students with visual impairments

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Introduction

Mathematics presents unique difficulties in the accessibility of content for students with visual impairments which are often little understood and so can be overlooked. This paper presents the findings of a qualitative study exploring the experiences of students with visual impairments and the perspectives of the staff that support them.

Participants

Data collection took the form of face-to-face, semi-structured interviews conducted with staff and students from four UK universities. These universities were chosen using a purposive sampling approach whereby institutions that were thought to be particularly illuminating or interesting were selected for study. Gatekeepers within these institutions, chosen from the authors' existing contacts, identified initial participants and further participants were identified using snowball sampling. Interviews were audio recorded and anonymised written transcripts were produced. In this paper, university names are removed and participants' names have been replaced with pseudonyms. Staff job titles and names of departments and subjects have been replaced with generic equivalents.

At University A:

- Jim - undergraduate student and Braille reader studying a mathematics-based subject in joint honours with a non-mathematics-based subject;
- David - undergraduate student and Braille reader studying a mathematics-based subject;
- Lucy - PhD student from a mathematics-based subject working as a part-time disability support worker;
- Susan - disability service adviser and alternative formats service manager.

At University B:

- Jackie - disability service adviser and alternative formats service manager.

At University C:

- Eric - alternative formats service manager;
- Sarah - mathematics support tutor working with students who had difficulties with mathematics for a number of reasons including some with visual impairments.

At University D:

- John - former undergraduate, now postgraduate, student and a Braille reader studying a non-mathematics-based subject with a mathematics component;
- Hazel - disability support worker involved with making documents accessible to John and other students.

Accessing mathematics through alternative formats

Many alternative formats – e.g. Braille and speech – are linear, and this can cause issues. Hazel felt it was difficult for her students to "understand the layout and structure of mathematics" from the linear representation. She said, *the student will ask me to read it out even though the [Braille] resource is being produced... once the student has an idea of the structure, the layout, where everything fits in ... he can go back to his own resource.*

Hazel explained the need to produce a resource which can be read quickly, since this may be used in a situation where the student does not control the pace of events, for example: “in a lecture room environment it’s not as if they can just stop and focus on ‘what does that formula actually [say]’.”

David reported that having the English text of an exam paper read to him was quicker than Braille, but it was necessary for him to “look at the mathematics on the Braille copy for any fine details of any equations.”

Many different standards exist for Mathematical Braille and Sarah found Braille was being produced in a different format to that with which her student was familiar. As an alternative to linear Braille, David was not convinced by two-dimensional Braille, saying he didn’t like “how it actually moves the Braille character... and also it sometimes will cram letters up... a bit closer than they should be”. However, he felt that if he had used this system from the start he, “probably would get used to [the differences]”. He said that a student new to Mathematical Braille should give “good consideration” to using a two-dimensional system, but be aware of its drawbacks, such as the difficulty of manipulating mathematics in this format.

David found that as the mathematics advanced beyond A-level he came across symbols he didn’t recognise and had trouble relating what was said in lectures to his notes, or reading aloud his work for a scribe.

Instead of “the whole hassle” of Mathematical Braille, John received materials in LaTeX and worked with Hazel to turn these into a form of human readable TeX (e.g. without the LaTeX formatting commands). John considered this to have two main advantages: this avoided the potentially error-prone and time consuming process of conversion; and, by reading the LaTeX, he was learning to write his own mathematics.

When enlarging mathematics for a large print format, Hazel found that LaTeX sometimes put insufficient spacing between symbols so they were printed too close to distinguish. Another issue for Lucy producing large print and John reading Braille is breaking mathematics over several lines. John said “if you break equations in the wrong places you get the wrong idea”.

A computer voice interface allows greater independence, but a sighted assistant will be better placed to deal with misconceptions arising from ambiguity. Hazel said,

sometimes [lecturers] say completely different things to what they are writing down so they might say ‘a plus b over c’ that could be ‘a plus b all over c’ or that could be ‘a plus stop b over c’.

However, Hazel felt a human reader was not always adequate because a printed resource “is permanent twenty four hours a day which I am not... even if you record [my description], you want to explore the formula in different ways next time.”

To access graphics, participants used a mixture of describing graphs and providing tactile diagrams, depending on complexity and student preference. David had diagrams described if they were “simple enough,” while Eric only used this alternative to tactile diagrams on “very rare occasions where the diagram would be so complex as to necessitate a verbal description.” Jim felt descriptions were not useful as,

what the [describing] person sees... as important, is not necessarily what [I] see as important. And they tend to either miss out features that [I] need, or just give far too much detail.

The original graphics may need to be simplified to produce a tactile version. Sarah found she needed to produce several versions of a single graph, saying,

I would do a separate tactile diagram with just the first line, and then do a second one with the second line and then do one that puts the two lines together. So we can build up the picture because I find that the diagrams he comes with are actually quite difficult if they have multiple *lines on. He tends to jump between lines quite often so it's useful if we can build up each as a separate entity and then start looking at their interactions.*

Some respondents worked with a sighted reader who would operate mathematical software which was inaccessible, sometimes converting software output into an alternative format. Some found alternatives to standard software that were more accessible but use of alternatives was not always permitted in assessments.

David reported frustration at not being able to pick up a textbook in the library and flick through it to see if it is relevant. He was helped by library staff, but as non-subject specialists, "they didn't really have any knowledge of what they were reading." David said,

If I could see to read print then I would go across to the library and look for a suitable book that seems to cover the areas and have a read of that and see if that explains it.

He felt that, had it been possible to have a Braille textbook produced, he may have found that he didn't "get on with that author or textbook," so once he had got the textbook he may "think it's a waste of time."

Jim was told his university could not "find any Braille mathematics textbooks," and as a result he was told he "strictly only [needed] the notes" from lectures to study the course. David expressed his frustration at not being able to read around the subject: "I know that I'm probably capable of high marks but I'm just not able to do it 'cos I haven't got the knowledge."

Conversion of materials to alternative formats

Jim found that adapting his non-mathematics module material was,

easier, because it is just purely texts, and now I don't actually rely on the support at all,... I just scan all my books and read them on my computer... [The mathematics based subject] is a bit more difficult because of all the mathematics symbols and all that, and sometimes graphs and diagrams.

Hazel explained that part of her job was "badgering" lecturers for reading lists and lecture notes as far in advance as possible. She said,

There is a lack of smoothing across the academic year... We make every attempt to foresee this... before an academic year, we try and get the reading lists in at the moment they are ready... but if the lecturer hasn't written the course yet, there's not much you can do about it.

At times, this information comes in very late and Hazel said "there are times when you are working late because a student needs a resource [but] you hadn't got what you needed to produce it in time."

Hazel supported students in modules where notes were not available. Sometimes the lecturer had outline notes they worked from during lectures. Hazel met with the lecturer to try to find out how closely the notes reflect what is happening on the blackboard. She said,

This is the worst problem, trying to take a resource that wasn't meant to be seen by student's eyes and turn it into something the student can use during the lecture to follow what is happening on the blackboard.

If notes cannot be made available these can be taken by hand during lectures by a support worker, and then converted into an appropriate format. David said this usually took "a couple of weeks but at one point it got really bad last year and once I didn't see a set of notes till six weeks after the lecture." David received additional tuition but felt he was not able to use this to "full potential" because he didn't know whether to talk about the lecture he had just attended, or the notes he had just received from a previous lecture. For one module Jim got none of the module content in Braille until three weeks before the exam. He said he felt "the whole term was... wasted 'cos I... couldn't do anything for the module. And in that three weeks I just had to revise, read up and learn."

David found there were sometimes errors in his typed notes, but said,

> *if [a mistake] has been introduced by the typist typing it up I may not remember the lecture well enough to know that doesn't quite make sense to what was said in the lecture because it was so long ago.*

David found errors were introduced by typists and by Braille transcription software "just not working very well." He found this a problem because "in an equation just one character can make a big difference to the meaning." John said this can mean "you often miss the point of what's being said."

David asked about proofreading and was told "it would just be too time consuming." As a result, he felt his Braille notes could not be completely relied on. He said,

> *There is a potential that I may have something missing that I don't even know is meant to be there and it just adds on an extra layer of thought that I've got to give of checking the notes as well as understanding them.*

David felt that "even just to do a random sample of documents would be better than nothing."

John had trouble getting editable versions of textbooks. At the start of the year he received a reading list and had a "panicked time" trying to contact the publishers for electronic versions. He said even when a lecturer only expects students "to read a few pages here... at times a section", it is still necessary to request the whole book.

The process of contacting publishers was time consuming. Whether they reply with an electronic copy or to say they cannot provide this, John had "never come across a case where it has taken us less than two weeks" and the longest saw the textbook requested before the start of the academic year arriving just after the January exams, six months later.

A possible solution was contacting the author directly. Hazel said "the authors range from completely ignoring us to being extremely helpful and sometimes... give us their own electronic copies of their source" or "email their editor and we need that because the 'Permissions Department' at publishers... don't know anything about LaTeX source files."

Many documents are now available as eBooks. Hazel explained that the mathematics in eBooks is often images and cannot be accessed.

John warned that not all electronic formats are suitable for conversion to alternative formats; for example, he cannot access mathematics encoded in PDF textbooks and PowerPoint lecture slides. He said his lecturers are, "aware that 'you know next semester John will come along to my course so I'd better not have any PowerPoints'." When provided with an unsuitable format, Hazel explained she would,

> *Scan it using standard English [Optical Character Recognition (OCR)] to get the English. The mathematics comes out as complete gobbledygook and then we use human assistance to deal with this... we put the mathematics in by hand [as LaTeX].*

The expertise with specialist software may lie with the department and not with the disability support service. As some time had passed, Jackie no longer felt confident using the mathematical typesetting software on which she had previously trained.

Receiving a LaTeX copy of a textbook was not the end of the story as for John. Then, he said, "the whole task of processing [into human readable TeX] starts and once that is done you have to start reading it as well."

Teaching and assessment

David reported confusion when lecturers gesture at the board and say "this term." However, he said, one lecturer "did read out everything and the equations just got too long and I lost concentration half way through and I couldn't keep track." Jim said,

> *first, it's hard to follow a long equation, even if it's read out, and second, they don't always remember, which I don't blame them. But even if they read it out I don't think it's gonna be*

that helpful, you know, you have about a page long of proof which even if they read it out you're not gonna follow in your head.

David felt a compromise was needed, "so that it makes sense without being able to see the board but not necessarily reading everything." Having the lecture notes in advance gives something to follow during the lecture.

When partial notes were used with blank spaces for students to fill in lecture content, Hazel produced two sets of notes,

one [with] an outline space where the student can write in... The other version [has] what's going to appear on the blackboard so the student... can copy from what's on there if it is useful for them to write down everything in the same way as everyone else would.

If notes cannot be produced, students may bring an assistant notetaker but may get little out of the experience personally. Jim didn't find listening to lectures he couldn't see rewarding enough to attend his mathematics lectures.

Support workers act as sighted readers, scribes, amanuensis, library browsers, tutors, notetakers or in a general support role. David felt it was necessary to take time to build a working relationship with the support worker and it was important the worker was familiar with the subject matter. Sarah felt it was important for the student to have the same support worker in each role and said,

the relationship between them is I think a key aspect of what we do... It's about communication and setting up effective communication between the two people involved so it's very much working with the partnership.

Interaction with a support worker and extra time taken to read Braille meant David received 50% extra time in exams and he felt this was necessary and sufficient for his needs. John, however, tried not to ask for extensions. He said,

Of course that puts... the pressure on something else so for example... I have to produce something by Friday, if I postpone it to next Friday, it will mean that whatever I was going to do in the second week is going to be pushed even further and it would be kind of an escalating effect.

Jim also tried not to ask for extensions. He said,

I don't see any point in having extra time. I take it as a given fact that if you have a disability you just have to work that little bit harder maybe,... I mean you can't ask for an extension of your life.

Nevertheless, it is worth asking whether it is fair that students have to complete more work in the same period of time. Some respondents reported handing in incomplete work or work they knew to be incorrect in order to keep up with the deadlines.

Conclusion

Students with visual impairments may use a number of methods to access mathematical content, and may have very specific alternative format requirements. Whatever a student's chosen method of accessing materials, problems can arise in producing this format accurately, reading it correctly, manipulating mathematics, accessing graphics and producing written work. Students may require adaptations to teaching and assessment and need alternatives to the standard equipment and software that are generally available. Despite all this, evidence suggests that students with visual impairments can do well in mathematics if the right support is in place. John said "if I didn't have Hazel helping me out here honestly I don't think I would be where I am."

Acknowledgements

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Mathematics for visually impaired students at A-level and the transition to degree level

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Introduction

I have been Head of Mathematics for over 25 years at New College Worcester (NCW), formerly Worcester College for the Blind, a secondary school for visually impaired students. The visual nature of some aspects of Mathematics has always posed difficulties for some of my students but others have found no difficulty in visualising situations which would have me reaching for a pen and paper to draw a diagram. One of my earliest memories of teaching a blind student involved trying to work out the height change in a bifilar pendulum when it rotated through a given angle. The blind student had completed the solution whilst I was still trying to draw the diagram!

Over the years I have taught A-level Mathematics and Further Mathematics to a number of students who have continued to study Mathematics, or Physics at university. Until recently I have always been concerned as to how those students using Braille would be assisted through their degree course. In this article I outline some of the issues for visually impaired students but I explain why my concern for those using Braille at university has recently been eased.

A-level for visually impaired students

There are no reasons why visually impaired students should not study mathematically based subjects at A-level. The barriers are, to some extent, less than those at GCSE level where there are far more diagrams and topics where vision is a considerable help. The majority of A-level topics are less diagrammatical in nature but there is still an issue of time as the coverage of some topics is likely to be slow.

Graphical calculators are a potential problem as they have been allowed in A-level examinations for a while. So far the only questions I have seen that require the use of a graphical calculator have been in sections where there is choice and another topic can be answered instead. MathTrax [1], software from NASA, can be used by blind students to give an audible tone for a graph but it is not at all comparable with the many features of a graphical calculator. Graphing packages on laptops can be used by partially sighted students but I do not know of any examination board that allows these to be used in examinations.

Partially sighted students and mathematics

Partial sight has many variants and each student needs to be treated individually. Students may be able to handwrite but many partially sighted students find that using a laptop enables them to produce material that they can read more easily. Students should consider using a word processor that can also deal with mathematical symbols. MathType [2] can be used either in a Windows environment or a Mac OS X environment but I only have experience of using it in Microsoft Word where it works well. MathType allows the creation of keyboard shortcuts and initially I spent time creating numerous ‘in-house’ shortcuts for students to learn. However, as these shortcuts needed to be repeatedly re-entered when student laptops went wrong, I now teach the keyboard shortcuts built in to MathType. These shortcuts are well worth using and it is easiest for students if the shortcuts are taught when new symbols are first introduced.

Whilst a graphical calculator may not be clear enough to see, there are software packages that enable partially sighted students to access similar functions for classroom use. At NCW we have used the software Autograph [3] which has the facility to create a default of thicker lines for graphs and grids.

Enlarging a text book on a photocopier is a quick and easy option but it rarely produces good quality material as the original has not been prepared with visually impaired people in mind. Students should ideally be provided with good quality large print materials in either printed or electronic format.

Blind students and mathematics

The medium of Braille continues to be suitable for those who are likely to take Mathematics only as far as A-level. All topics can be covered, although without use of a graphical calculator. Some topics are slow to cover; long algebraic equations can be cumbersome in Braille and matrix work is a bit awkward to set out, but it is all possible. Three dimensional work may require the use of models and sketching curves is often done by a finger on a desk, rather than using drawing film, simply to save time! Pre-prepared tactile diagrams can be used for most diagrammatic work but the original diagrams may need to be simplified to enable the salient points of the diagram to be clear.

A few blind students who take A-level will be intending to continue on to a degree course that has a mathematical content. These students should seriously consider using the method that Alastair Irving and Robin Williams started to develop whilst studying for their A-levels at New College Worcester (I will call this the ‘A&R method’ for brevity) [4]. By making use of various software packages, a Braille strip, good computer skills and initially considerable time and effort, they were able to produce mathematical work of a high quality in print. Their proficiency in the method was such that they were able to take their external examinations independently and produce a print script for the examiners to mark. They were able to be sent material in Microsoft Word and MathType, change it into LaTeX [5], modify the LaTeX to create a version that they could access in either speech or Braille (using a Braille strip attached to their laptop) or more usefully a combination of the two. When they started to develop this method Alastair and Robin were both confident touch typists with good IT skills and they were good mathematicians. Since then they have continued to develop and make use of this method of working to complete their degrees and are now following postgraduate studies.

Unfortunately there is no world-wide uniformity in the way that mathematics is written in Braille. The Nemeth Code [6] is used in the USA and it is different from the Braille Mathematics Notation [7] used in the UK! As the A&R method was developed my input as the teacher was mainly in providing the initial materials and in checking the final output. Alastair and Robin dealt with the bits in the middle and they considered that the Nemeth Code had advantages over the UK code. Any student learning to use the A&R method has this extra hurdle to deal with, although I have to say it does not seem to have been a problem for those who have used it so far! However, it is because of all the additional learning that needs to take place that I consider it is really only a suitable way of working for a student who is intending to do a subject involving mathematics at university. For these students I believe the method creates considerable benefits in higher education and therefore makes it well worth learning and I would suggest that it is essential if they are to have the level of independence that they deserve.

Transition to Higher Education

Students start university with considerably different experiences and all visually impaired students need to be treated individually. University staff, probably at departmental level, should invest time before the start of the course to understand the problems involved. The issues surrounding a technical subject, such as mathematics, are more complex than those for a purely text based subject. In addition to teaching a particular subject a department may need to teach visually impaired students methods of access and various codes during the course.

The issues for partially sighted students are likely to be fewer in number than for those working in Braille but it important for departments to have a clear understanding of the methods that students use to access and prepare material.

Some Braille users will have had a learning support assistant who has underwritten their Braille for their sighted teacher to read. This means that communication between teacher and student will, of necessity, have been mainly oral. It is possible that students will have created their own symbols in Braille and not used the correct code according to the Mathematics Braille Notation. Other Braille users will have been taught by a teacher who knows the Mathematics Braille Notation and they should have a good knowledge of the code. These students will be used to having their Braille work marked by someone who can read it and they may expect this to happen at university. All Braille users are unlikely to have met all of the Braille notation that they will need to use in their degree course and the university should consider how new elements of the code will be dealt with during the course. There will be a certain amount of adjustment for all blind students when they start any degree course. It is likely that an individual student will unaware that the way they have worked at school may not be transferrable to university. It is advisable for there to be close links between any university and the student's school before the start of the degree course and this contact should continue during the first few months of the new course.

The likelihood of a blind student being supported in higher education by someone who knows the Mathematics Braille Notation is close to zero whereas there is a reasonably high chance of someone in a university mathematics department being familiar with LaTeX. This means that any student who is able to use the A&R method should find that they are able to receive material from their lecturers in a format that they can access and they should be able to provide their lecturers with material that their lecturers can read. The benefits of using the A&R method are considerable for both student and lecturers and providing the opportunity for students to learn this method before the start of a course appears to be in the best interest of all concerned. It may well be worthwhile investigating if funding for such training is available through the support grants for disabled students.

Lecturers will still need to adapt to having a blind student in their class in the use of language; they do not need to stop using 'sighted vocabulary' but they do need to be precise. 'Moving x from here to here' is utterly useless but saying 'look at equation 1' is fine as long as the braillist has a copy of the material with equation 1 clearly labelled! Materials obviously need to be provided before the start of lectures in a suitably accessible format.

The future for the A&R method

Learning to use the A&R method is an 'extra' for students whether it is learnt alongside A-levels or later but it brings with it a level of independence that should be the right of any blind student. The advantages of it should be known about at school level and throughout academia but current knowledge of it is limited and its documentation is sparse. Whilst I assume that the A&R method could be further developed into a simpler single package its specialist nature means that it will always be used by only a small number of students so I doubt if it has any commercial viability. Its use will always be limited as the number of blind students studying mathematically related subjects at university will be small. However, it provides a significant breakthrough for those who use it and deserves as wide an audience as possible.

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[4] Alastair Irving and Robin Williams have a separate document in this publication, pp 19-23.

[5] http://www.latex-project.org/

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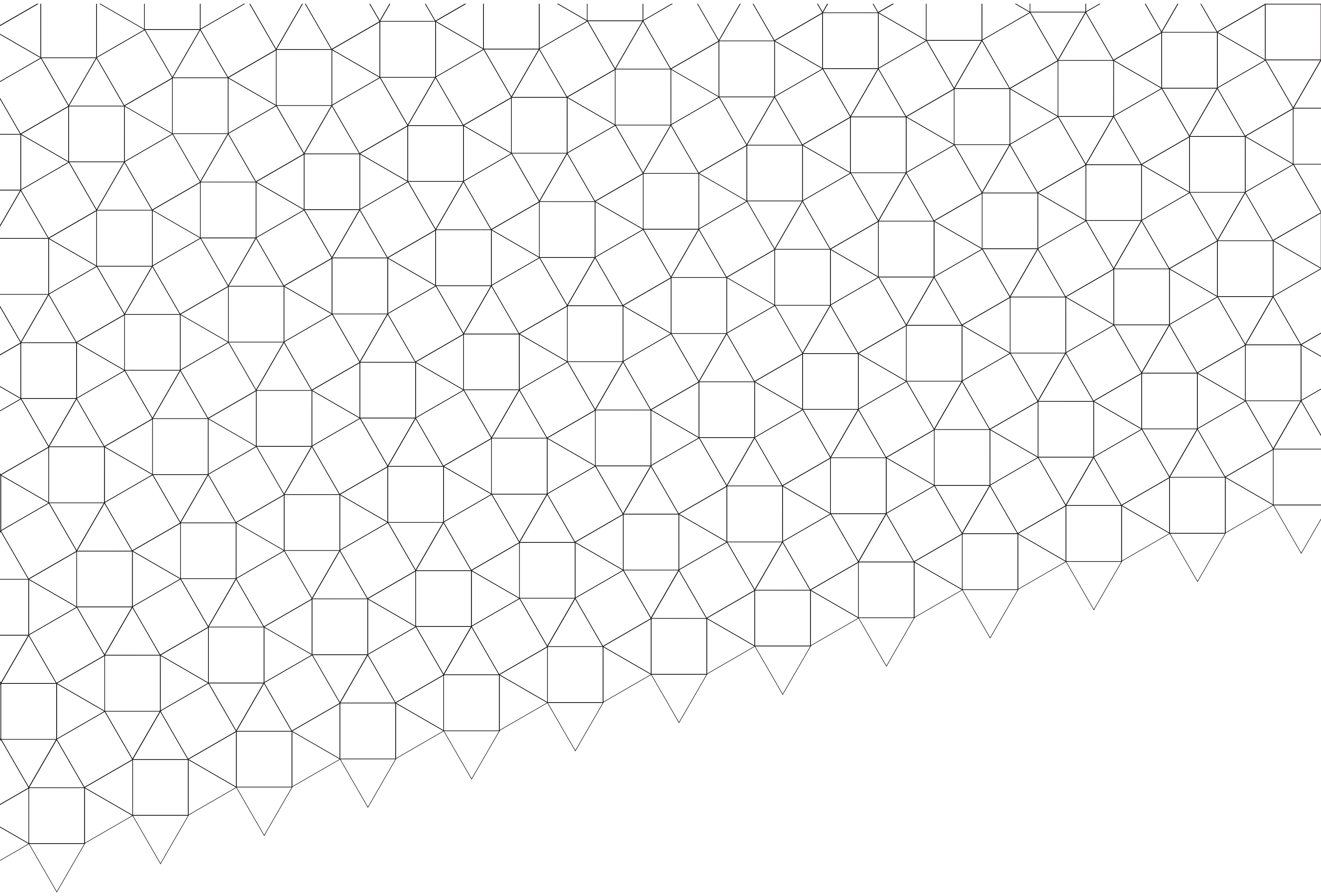

On the accessibility of mathematics to visually impaired students in higher education

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Introduction

This report is a case study of the methods used by the authors to overcome barriers to the accessibility in the study of mathematics at the higher education level. By employing the methods documented here both authors achieved class I Masters mathematics degrees from their respective institutions, and are now pursuing research activities towards PhDs (one author in probabilistic weather forecasting, the other in analytic number theory).

It should be noted that we focus largely on those students who have no sight or minimal vision. It is felt that students registered as partially sighted should face far fewer problems when dealing with the issues outlined below.

In the following we outline the issues we perceive as the main barriers to the study of Mathematics, Statistics and OR (MSOR) subjects, with particular focus on the higher education (HE) level, and describe the methods practised by the authors to combat these barriers. As this report is merely a case study of the authors, few references are listed. The authors have developed large proportions of their working methods on an ad-hoc basis as necessary, guided at younger ages by David Spybey[1].

Assistive technology and its implications

It first helps to outline the various forms of assistive technology available to aspiring Visually Impaired (VI) students. We describe only a small subset of the raft of available technology.

- **Screen reading software and refreshable Braille displays**

  A screen reader is software capable of converting output printed to the monitor into audible speech. Modern-day screen readers typically provide access to the vast majority of common applications. However, there is still a lack of support for mathematical documents such as those written in MathML[2] [1], and Microsoft's Equation Editor.

  Refreshable Braille displays are hardware devices which typically produce the screen reader output on a Braille line, by means of piezoelectric cells. The vast majority of displays provide only a single line, although there are now some two-line models. The length of displays varies considerably. The expense is considerable.

- **Braille embossers**

  A Braille embosser is a hardware device which prints documents in Braille. Typically translation software is required to convert the regular text to regular Braille code before printing. Whilst many translators are available for standard text documents, there is again a lack of support for scientific documents. Most translators are incapable of producing scientific Braille from a scientific document, regardless of the format supplied (PDF or LaTeX for example).

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[2]Please refer to Copper, pp39-45 of this collection for an explanation of MathML.

One exception is the Duxbury translator [2], which is capable of producing Braille output from a supplied LaTeX source file. At the time of writing, anecdotal evidence suggests that the quality of translation varies with the supplied document.

Writing such translation software is no easy task. It is equivalent to writing a complete LaTeX compiler in which the output is Braille, rather than the standard DVI, PS or PDF formats. To write a LaTeX-to-Braille translator would require the capability to handle cross-referencing of equations and tables, handling of a large variety of nonstandard packages and their specific commands, and so on.

- **Older, more simplistic methods**

  There are many methods which whilst being somewhat simplistic are nevertheless of practical benefit.

  o German film is a plastic substance which when placed on a rubber mat produces raised lines when drawn upon. We consider this very useful when a quick sketch is of use. We use this material when having informal discussions with colleagues, and when necessary to understand diagrammatic content of lectures at undergraduate level. The material is available from the RNIB [3].

  o Zychem [4] sell machines which enable the production of embossed diagrams. These are appropriate when a higher quality, robust diagram is required, such as in exam situations. Swell paper is printed upon and passed through a heater which forms embossed lines.

  o Mathtrax is software available from the NASA website [5] which depicts graphs by means of an audible tone. The user enters a function of interest and can then listen to the `soundscape' of the graph, the tone changes in pitch depending on the value of the function, and moves from left to right along the x-axis. We made fairly frequent use of this software at undergraduate level, although much less so at postgraduate.

Accessing reading materials and submission of written materials using LaTeX

The most common problem known to the authors facing MSOR undergraduate students is in the obtaining of accessible reading materials, and in the submission of written work. Here we outline the issues at hand.

1. Obtaining accessible mathematical documents

Scientific documents are by their nature nonlinear in their format. A simple example is the two-dimensional form of a fraction. As severely VI people, we must rely on speech and Braille output, and are therefore confined to work in a one-dimensional space. Even with the most up-to-date technology available, we feel there is no satisfactory solution to dealing with this nonlinearity other than by means of a linear representation.

Intuitively, the most natural means of reading MSOR documents would seem to be Braille. Braille is a linear format with a sophisticated mathematics code. However as described in the section on assistive technology and implications, acquiring properly translated Braille is a highly complicated task which is not yet possible from a source file such as a PDF copy of a textbook.

We therefore use a suitable replacement in LaTeX, using the actual LaTeX source code. The source file is linear in its nature, and is thus suitable for use with assistive technology. The user is able to read and interpret a scientific document using speech output and / or a single-line Braille display.

If the user needs to read a document of any length or complexity, LaTeX source code is somewhat complicated and long-winded to be read in its raw form. We have therefore developed a set of script files to process a line of LaTeX code into a more audible rendering of the mathematical content, as well as to an accurate translation of that particular line into Braille. This system, which we have named LaTeX-access [6], is described below.

The most important reading materials to be obtained are lecture notes. We highly recommend that lecturers be encouraged to produce a copy of their notes in LaTeX, whether the source file is written to produce slides, notes to be handed out to the entire class, or simply notes which the lecturer refers to during the course of the lecture. The student can then follow the lecture by referring to the notes on a laptop computer, rather than having to spend time catching up afterwards. We found that significant benefits in learning were gained from courses in which well prepared notes in LaTeX were available, and in which transcribers were not required to type up handwritten notes into LaTeX after the lecture. Indeed, there is a significant correlation in one of the author's exam results between those courses in which good notes were available and those in which transcription services had to be used. In any case, having well prepared notes is good general departmental practice.

2. Obtaining accessible diagrams

Sometimes it is necessary to obtain accessible copies of diagrams, which may for example have been drawn on a blackboard by the lecturer or form an important part of a textbook. This can often be a time consuming and somewhat haphazard process, and so we recommend that careful consideration be given to the selection of those diagrams which are really necessary (many diagrams are just used to increase the visual appeal of the material and are not needed to understand the mathematics). Depending on the quality of diagram required, a simple sketch on German Film or use of a ZyFuse machine (see above) may well suffice. Alternatively, if the diagram is available as a computer-generated image, a Braille embosser could be used to print a higher resolution diagram. It should be noted that tactile diagrams are limited in their resolution. We strongly recommend that work be carried out to reduce high resolution diagrams to show the most important features before embossing, diagrams of too high a resolution are likely to be indiscernible to the user and of no practical use.

3. Using LaTeX to submit written work

A common theme in the discussion of the participation of VI MSOR students is in the submission of written work. In the past, undergraduates have often relied on somewhat archaic methods of producing Braille documents by hand and relying on a transcriber to produce a printed version. However, we strongly feel that such methods must be consigned to the past. Such services take considerable time and effort, often at additional expense to the institution. Moreover, they limit the student's independence of study and collaboration with their peers.

We strongly believe in the use of LaTeX for the production and submission of written work. This method enables the student to work independently and at a time of their convenience, and hence on a par with their peers in terms of the opportunities available to them. Using LaTeX, the student can submit a well-produced printed document, whilst being able to work with speech and Braille by using the LaTeX-access scripts. This method of working served us well at undergraduate studies, and continues to do so at the postgraduate level.

The LaTeX-access scripts

The LaTeX-access scripts are written in the Python language and are available from [6]. The scripts translate the line of LaTeX at which the cursor is located into a more audible rendering of the mathematics (which is output by the screen reading software), and into a Braille translation (which is output on the refreshable Braille display).

The scripts enable easier reading of a variety of MSOR features written in LaTeX. On a simple level, many symbols (such as Greek letters, statistical and group theory notation) are translated into speech and Braille. For example, fractions using the \frac command are translated into Braille and spoken as 'a over b', where a and b are the numerator and denominator respectively. Subscripts and superscripts, as well as formatting commands such as \mathbf are accounted for both in Braille and by a change of pitch in speech.

On a higher level, matrices can be processed such that the user can read up and down columns or along different rows without moving the cursor position, an improvement on what is possible with the raw source in which only a single line can be viewed at any one time. Similar functionality is implemented to enable easier reading of tables.

We stress that the scripts are not an attempt to perform a full translation of the LaTeX document, such as that described in the section on assistive technologies above. Rather, the scripts translate only the current line. They do not handle the more style-based features of LaTeX, such as cross-referencing of equations. The scripts cannot therefore be used to produce hard-copy Braille, such features are of no interest to us as we feel it to be advantageous to read and edit the LaTeX in real time.

The LaTeX-access scripts can be used in any windows-based text editor. Currently support is available for the JAWS screen reader, with work ongoing for support by the open-source screen reader NVDA at the time of writing. The scripts have also been ported to a usable level for use in the Emacspeak package under the Linux operating system.

Making a choice of editor

The choice of editor in which to read and write LaTeX is important. Some editors have accessibility issues, such as the latest versions of WinEDT under Windows. We recommend the use of Emacs with an installation of the AucTeX package for producing LaTeX documents. Emacs has no major accessibility problems and is extremely powerful. In particular, we make extensive use of the macros which enable keyboard shortcuts to be used for the fast production of common LaTeX code, this considerably reduces the time involved in typing the entire document manually. However the choice of editor is flexible and is a matter of taste.

Converting documents to LaTeX

Ideally, one would obtain materials such as textbooks in LaTeX by means of contacting the publisher and / or the author directly, however in some cases the document of interest was either written in another format or is too old for the original source files to be traced. In this case, the user has three available options.

- Attempt to find an alternative reading source which comprises the same information and was produced in LaTeX,
- Use a transcriber to write the most relevant parts of the material in LaTeX,
- Attempt to convert any files available from their format into LaTeX.

Infty Reader [7] is software written to enable the automatic conversion of MSOR documents into LaTeX. The user supplies a PDF of the document, or alternatively image files of the pages. The software attempts to convert the document into LaTeX (amongst other output formats). We find this software often performs well, although the supplied files must be of a sufficiently high resolution.

Summary

Here we have outlined methods which have enabled us to study mathematics at HE level and provide a good foundation for the continuation of our study at postgraduate level. Our thoughts come as recommendations only, and are not necessarily applicable to all students. It is however our strongly held belief that adoption of the use of LaTeX is the most efficient and accessible method of working for severely visually impaired students. We feel it is unreasonable and unnecessary to expect students to adopt LaTeX as a method of working at earlier ages, although we suggest that a student who intends to study an MSOR subject at the HE level should be advised to familiarise themselves with LaTeX before the start of their course.

LaTeX is also a highly suitable format for producing documents for people who are partially sighted, but who may require larger fonts or unusual colour contrasts. These can be easily set in the LaTeX source file, thus circumventing the need to involve lengthy transcription services.

An area of potential conflict is in the expectations of the department and institution from the student, and vice versa. We feel it necessary to stress that the student should expect to display a reasonable level of proactivity in dealing with their accessibility needs. After all, the student is likely to be the best person to understand the areas in which he / she has difficulties.

References

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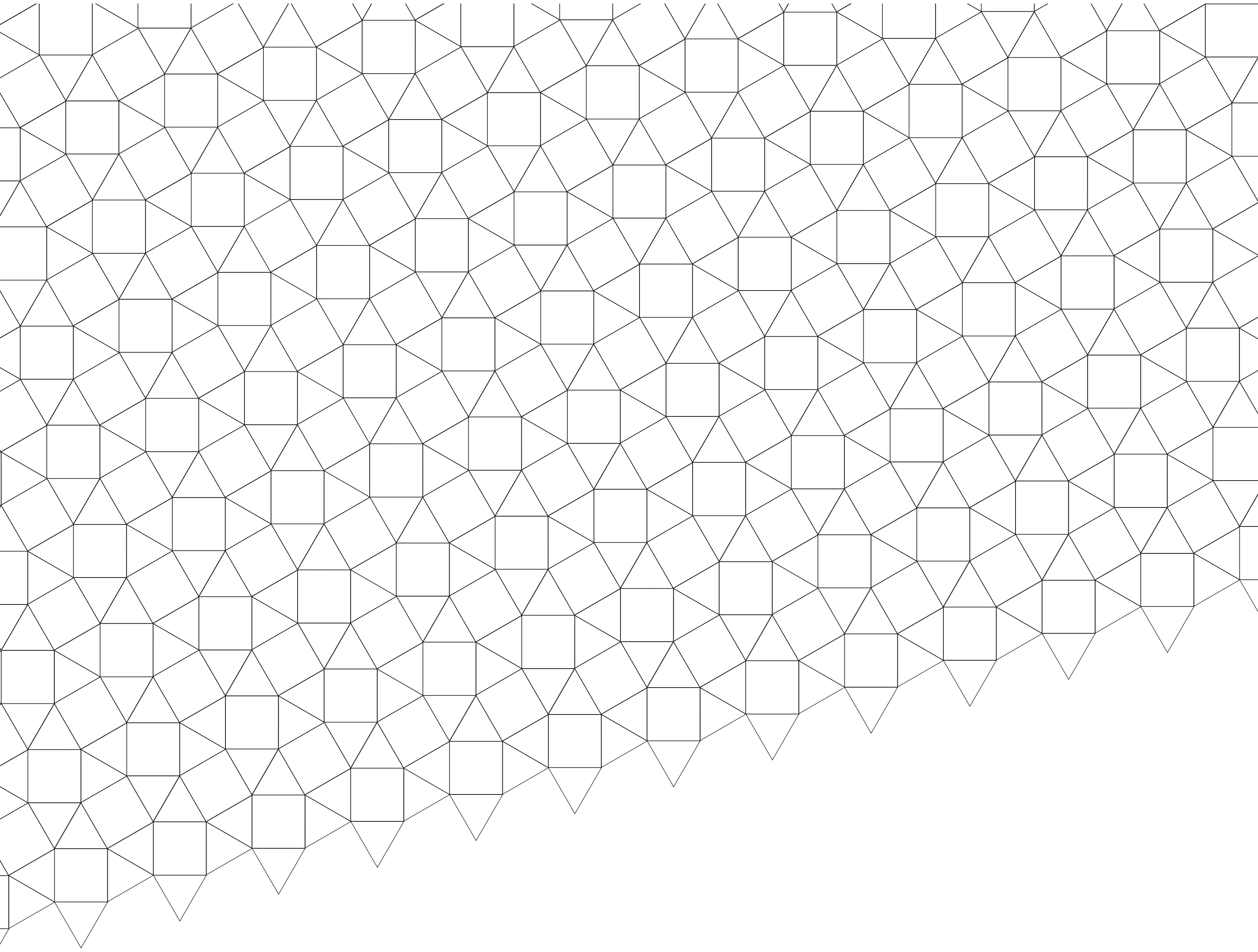
[3] http://www.rnib.org.uk

[4] http://www.xychem.moonfruit.com

[5] http://prime.jsc.nasa.gov/mathtrax

[6] http://latex-access.sourceforge.net

[7] http://www.sciaccess.net



Mathematics, dyslexia, and accessibility

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Context

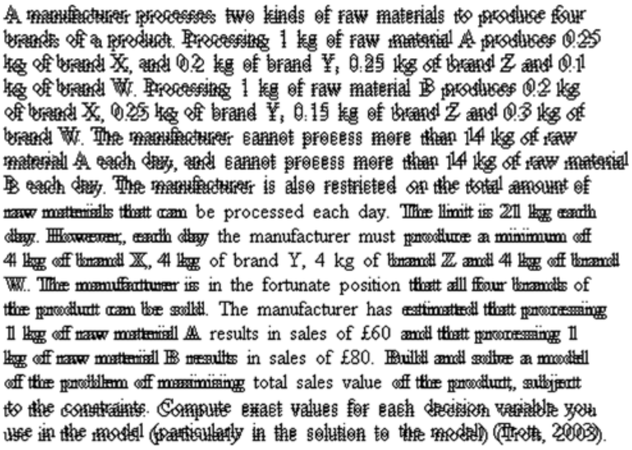
This paper is based on my experiences in one-to-one mathematics support to students with additional needs, in particular dyslexia. It will build on a number of case studies, in order to explore the differences that students experience and the errors they are likely to make. The aim is to show how greater accessibility could help the dyslexic mathematician focus on developing their mathematics and demonstrate their capabilities. Three areas will be covered, namely: reading, writing and memory.

"Dyslexia is a specific learning difficulty… It is likely to be present at birth and to be lifelong in its effects. It is characterised by difficulties with phonological processing, rapid naming, working memory, processing speed and the automatic development of skills that may not match up to an individual's other cognitive abilities. It tends to be resistant to conventional teaching methods, but its effects can be mitigated by appropriately specific intervention…" [1].

Barrier: reading

The first area to consider is reading. The dyslexic student may have difficulty reading connecting text or have a slower reading speed. Frequently, dyslexic students are attracted to the study of mathematics because it is perceived as requiring a lower level of reading text. One very able mathematics student, Joe, with high non-verbal ability, arrived at university with a very low level of reading, which had persisted from his earliest years. He was unable to read simple text effectively. Handouts and instruction sheets given out in lecture or lab sessions were particularly difficult for this student. Support work focused on identifying and understanding key words, symbols and procedures used in a variety of mathematical contexts, such as the statement of the tasks, examination questions or lecture notes. These key words enable a clearer focus in order to prioritise responses. It was also important for his academic tutors to provide material in advance of lectures and in the more accessible sans serif fonts, such as Arial, Verdana, Comic Sans or Trebuchet. For contextual text surrounding mathematics, Verdana is arguably preferable as the symbolic elements are clearer. Documents should be well spaced with clear headings. A coloured background helps to reduce the glare that many dyslexic students encounter through visual stress: "the inability to see comfortably without distortion and discomfort" [2]. Colour preferences can be individual and each student will respond to a different combination of coloured background, coloured text, spacing and fonts. It is important, therefore, to make all notes and handouts available for students to download and format as they choose. Figure 1 (overleaf) shows one of the visual stress affects ("whirlpool") that some students may experience and figure 2 (overleaf) shows the text in a more suitable format [3].

Another dyslexic mathematics student, Kate, had excellent numerical skill and a vivid visual memory; however, these strengths were offset by her weaker reading speed and accuracy, which only scored in the first percentile of the population. She also frequently lost her place when reading and had poor working memory. Support for Kate focused on the dissection and simplification of the more worded problems into a more manageable form. This included the use of bullet points to split text into smaller chunks, highlighting key information in two colours that distinguished the given information from the required instruction, e.g. to obtain a differential equation. It should be noted, at this point, that Kate had no difficulty at all with a page of mathematics almost devoid of connecting text, such as a solution to a differential equation.


A manufacturer processes two kinds of raw materials to produce four brands of a product. Processing 1 kg of raw material A produces 0.25 kg of brand X, and 0.2 kg of brand Y, 0.25 kg of brand Z and 0.1 kg of brand W. Processing 1 kg of raw material B produces 0.2 kg of brand X, 0.25 kg of brand Y, 0.15 kg of brand Z and 0.3 kg of brand W. The manufacturer cannot process more than 14 kg of raw material A each day, and cannot process more than 14 kg of raw material B each day. The manufacturer is also restricted on the total amount of raw materials that can be processed each day. The limit is 21 kg each day. However, each day the manufacturer must produce a minimum of 4 kg of brand X, 4 kg of brand Y, 4 kg of brand Z and 4 kg of brand W. The manufacturer is in the fortunate position that all four brands of the product can be sold. The manufacturer has estimated that processing 1 kg of raw material A results in sales of £60 and that processing 1 kg of raw material B results in sales of £80. Build and solve a model of the problem of maximising total sales value of the product, subject to the constraints. Compute exact values for each decision variable you use in the model (particularly in the solution to the model) (Troh, 2003).

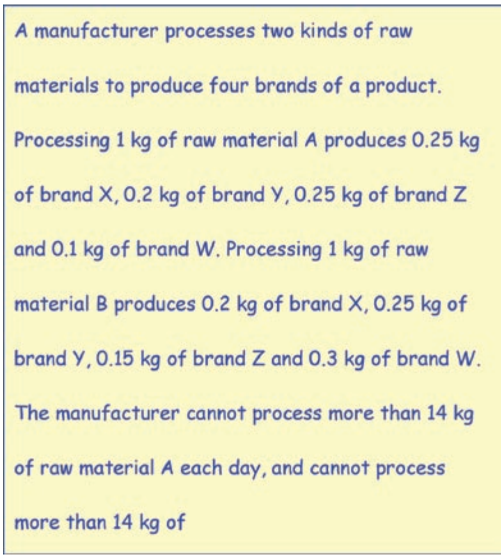

A manufacturer processes two kinds of raw materials to produce four brands of a product. Processing 1 kg of raw material A produces 0.25 kg of brand X, 0.2 kg of brand Y, 0.25 kg of brand Z and 0.1 kg of brand W. Processing 1 kg of raw material B produces 0.2 kg of brand X, 0.25 kg of brand Y, 0.15 kg of brand Z and 0.3 kg of brand W. The manufacturer cannot process more than 14 kg of raw material A each day, and cannot process more than 14 kg of

Figure 1: 'Whirlpool' affect

Figure 2: A clearer representation of the same text

Another particular issue for Kate was the use of mathematics course text books. Henderson [4] notes that in mathematical texts, tables, graphs and diagrams often intersperse text and each of these can have its own labels and subtext. Examples are often worked out in the middle of information or text on how to complete the calculation, so that if you have trouble reading the connecting text you may ignore both the text and the example. This irregularity is particularly an issue for dyslexic students. Kate found this to be the case for one book in particular that formed a core text. The interspersion of text and mathematics was high and many of the figures were out of order so that, for example, half way down a page it referred to a figure that was on the following page, or even a couple of pages further on. Kate would turn to the required figure, study it, but rarely returned to the point of the text. This meant that the sense was lost and the work without meaning.

Recommendations:

An essential requirement has to be the accessibility of text to students with dyslexia. Whenever possible the use of text books such as above should be avoided. Notes and worksheets should be made available beforehand and sans serif fonts used at all times. It is also useful for dyslexic students who find it difficult to read and interpret the feedback given on assignments, to have an MP3 recording of this feedback.

Barrier: writing

A second area that sometimes poses issues for the dyslexic is that of writing. Andrew was a physics student who excelled in non-verbal elements, had good number skills and sound reading comprehension. He was very articulate and a good physics student. However, he struggled to process at speed and had a poor working memory. His writing speed was assessed as being in the 5th percentile for the population. Andrew was holistic in his approach to tasks, working in a non-sequential way that arose from his intuitive feel for a situation. He was, above all, insightful. However, invariably this led to non-standard solutions and poor documentation of his method. His dyslexia also meant that he was inaccurate and made frequent errors. One example occurred during an exercise involving a series of row operations on a 4 by 4 matrix. His poor written presentation meant that the rows and columns were far from aligned so that the entries became confused and, having started with a 4 by 4, ended with a non-aligned 4 by 3. The use of centimetre squared paper was helpful in this instance. A second example occurred when Andrew was undertaking integration by parts of *cosh h cos h* with respect to h. The similarity of the two functions when written down caused errors in his writing and presentation of the solution. However, he was very clear about the nature of the problem and its solution. Andrew miscopied *cos h* as *cosh h* as well as the converse. Throughout, Andrew's written submissions did not have a logical order, flow or cohesion and there were frequent copying errors that occurred when transferring between media, pages or lines. Frustratingly, Andrew had an insight into the problems that enabled him to ‘see’ the potential for solution.

Recommendations:

Accessibility for dyslexic students should, therefore, include the willingness to accept less well documented solutions or facilitating the students to talk through their ideas and solutions. There should also be a willingness to accept sketches that are poorly defined (in the case of a dyspraxic student) and give a choice of diagram or description. It is frequently considered inappropriate to comment on spelling in essays and written work that comes from a dyslexic student; this would appear to be equivalent to the copying errors in mathematics. Fundamentally, it is about addressing the student’s understanding of the mathematics.

Barrier: memory

The third and final area for consideration in this paper is that of memory. Andrew had a poor working memory. It was important for him to draw on his visualisation skills in order to see situations. Indeed, Benoit Mandelbrot, the famous mathematician, had similar strengths:

> *“(Mandelbrot) was not a good student; it was said that he never learned the alphabet (he could never use a telephone directory, for example), nor his multiplication tables past five. Despite his poor performance at school, he found that he had a quite extraordinary ability to “visualise” mathematical questions and solve problems with leaps of geometric intuition rather than the “proper” established techniques of strict logical analysis. After the war he passed the entrance exams for the École Polytechnique, achieving the highest grade in Algebra by “translating the questions mentally into pictures””* [5].

Notation also contributes to the load on memory as students have to remember a wide range of standard notations throughout their studies. Michael Faraday [6] wrote:

> *“Mathematical formulae, more than anything else, require quickness and surety in receiving and retaining the true value of the symbols, and when one has to look back at every moment to the beginning of a paper, to see what H or A or B mean, there is no making way. Still, though I cannot hold the whole train of reasoning in my mind at once, I am able fully to appreciate the value of the results...”*

Issues of memory not only encompass remembering the symbolic notation needed, but also linking symbol, word and process. For example, in integration the integration sign, the word ‘integration’ and the process itself have to all be linked together. Students also need to frequently hold all aspects of a problem and various partial solutions in mind. An example of this is a partial differentiation problem. This type of question is most usually answered in a linear format, one partial derivative following another, and bringing these together later. For a dyslexic student, some of the earlier partial derivatives are forgotten and overlooked so that the final solution becomes intangible. By using a branching diagram, it is possible to create a situation whereby several partial derivatives can be kept at the same horizontal eye level. This eye-level is also an issue in scrolling because screen-sized information can be referred to, but if scrolling is necessary, then information from the top of the page will be easily forgotten as the scrolling moves down the page. See figure 3 (overleaf).

“Many students have significant difficulties thinking and reasoning about mathematical concepts” [7]. Often this follows from a definition or theorem. These definitions and theorems feature in many examination papers, often requiring students to state a particular definition or theorem, before going on to reason a proof. When statements of definitions or theorems (without proof) are assessed, it is a test of rote recall. One mathematics module contained 16 definitions and 42 theorems and the end of module examination devoted 24% of the marks to the recall of definitions and theorems, excluding the proof. While it is accepted that certain definitions and theorems are of prime importance, their rote learning appears to place at a disadvantage those students who find such learning difficult, but who fully comprehend the mathematics and who can develop the proof and utilise it appropriately.

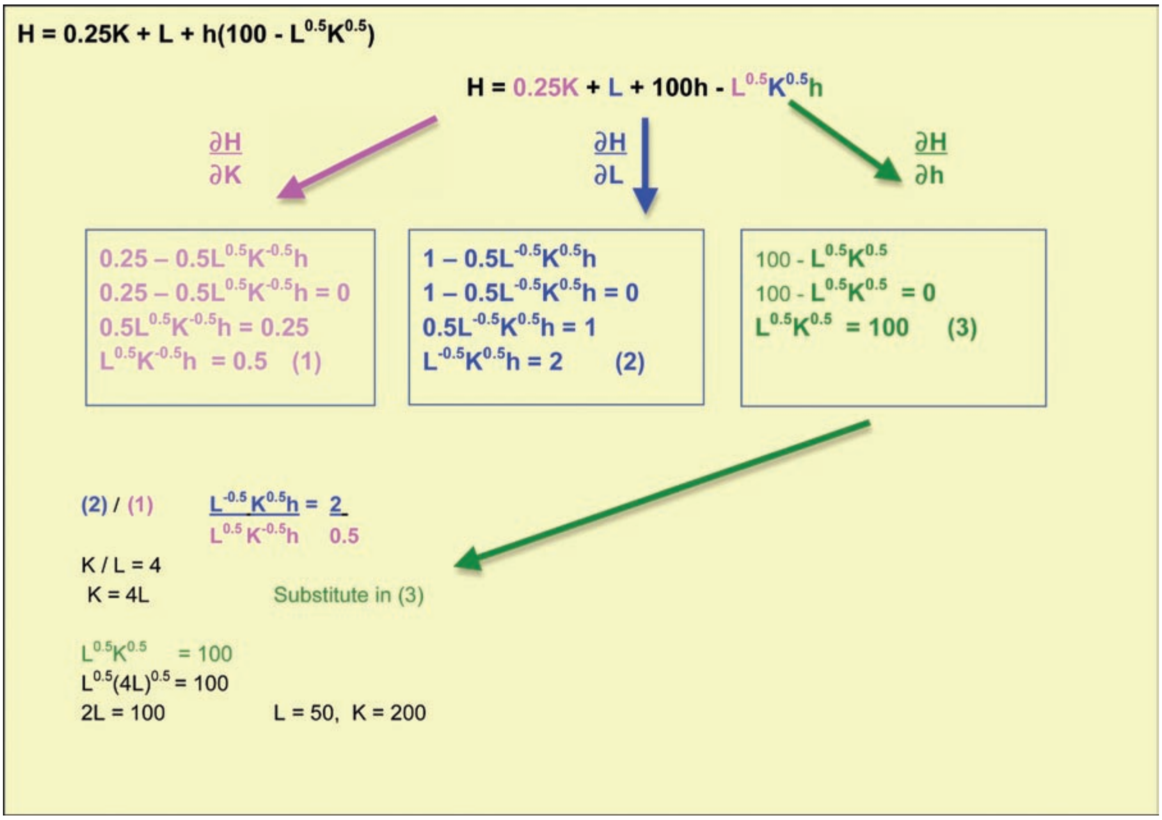


Figure 3: A “tree diagram” to show three partial derivatives horizontally

Recommendations:

Thus, such questions should, whenever possible, be avoided so that examinations do not rely on memory, and all necessary theorems and formulae are provided, together with a list of terms and notation used. It is also helpful to have consistent notation across modules and staff within a department. If there is a reduced load on the working memory, students can focus on understanding the mathematics.

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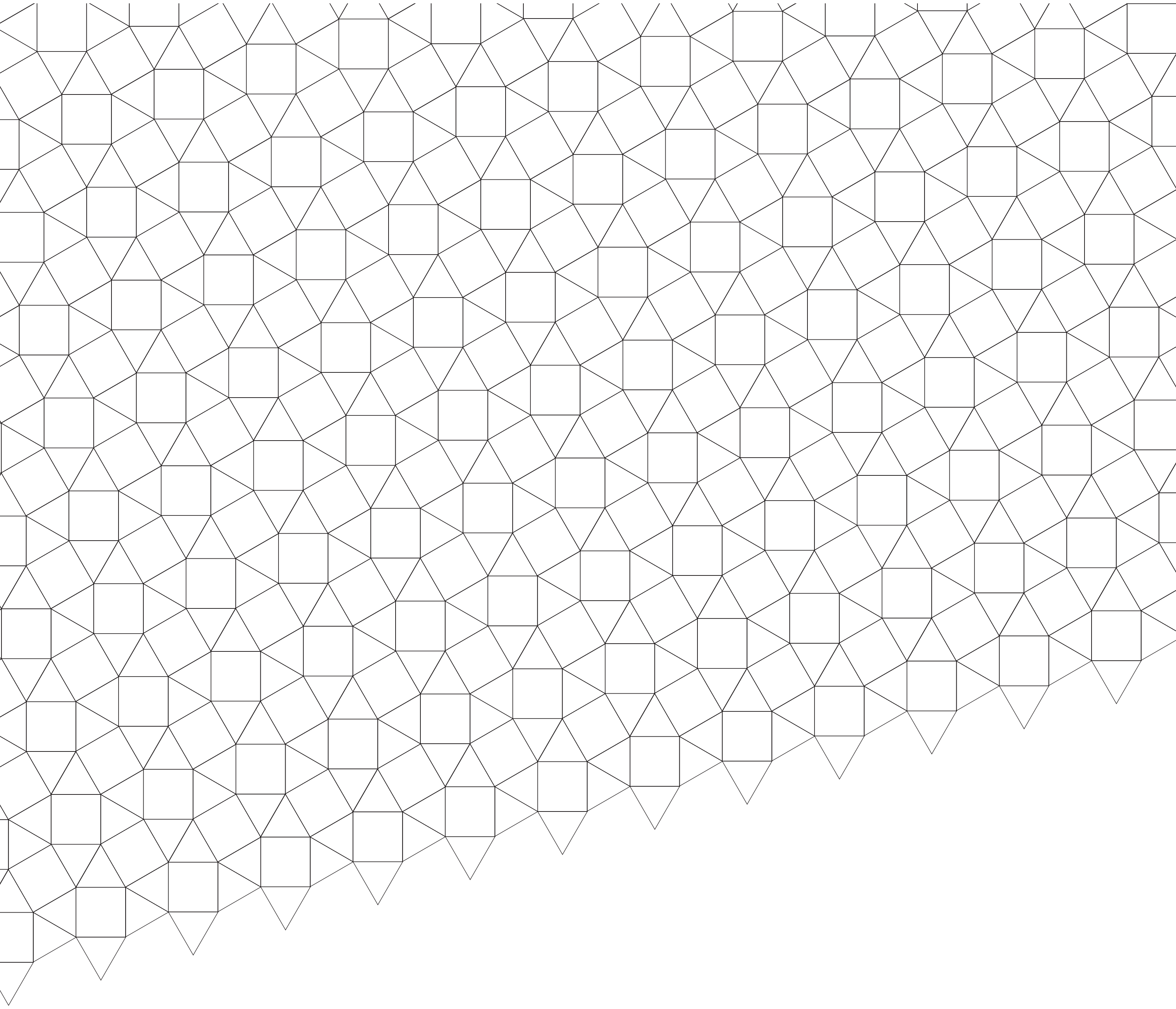

Assisting mathematics students who have Asperger syndrome

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Introduction

Asperger Syndrome (AS) is on the autism spectrum and associated with at least average intelligence [1]. This paper draws on experiences of students with AS to raise awareness of the challenges which might be associated with studying mathematics at university and suggests potential solutions. The REAL (Reliable, Empathic, Anticipatory and Logical) approach, which has been found to be most effective [2] is briefly introduced. This approach begins with excellent pre-entry support and advice, is mindful of the potential impact of all transitions and respects the individuality of every student.

Background

Martin [2], while cautioning against stereotyping, found the most typical student with AS to be male, in his early twenties, studying science based undergraduate courses, having achieved good A level grades. There is no such thing as a 'one size fits all solution' but some common themes have emerged regarding the challenges university may present, and about effective ways to assist learners to achieve success. Commentary directly from students is quoted throughout this paper (some of which is unpublished, for example, Madriaga et al. transcripts [3]). All necessary permission has been obtained and anonymity is protected.

Characteristics of AS

Baron-Cohen [4] describes the cognitive style of people with AS as usually systematic rather than empathic. Strengths are mainly around dedication, application, high levels of interest and motivation and often a logical approach [4,5]. Students who have AS apparently flourish in environments which minimise chaos and maximise order. Application and focus are positive characteristics associated with AS. Arnold reflects, from personal experience: '*Obsessive narrow interests can be valuable. Take computers for instance. Most people can just use them, but I can strip down and rebuild mine.'* [5, p.2]. Examples of very able mathematicians and scientists thought to have AS pepper the literature [6] although this can contribute the 'genius pressure'.

Typically challenges relate to social inclusion, communication and flexibility of imagination; sensory overload is also often reported [7]. The world can seem rather confusing when other people are inconsistent and focussing on details rather than the bigger picture can sometimes generate greater feelings of security in an uncertain world. Confusion can lead to anxiety which can manifest in attempts to maintain consistency and in fear of change. Transitions, such as starting at university or commencing a placement, can therefore be especially troubling [3]. Seemingly little things can perpetuate apparently disproportionate responses mainly because of difficulty with the flexible application of problem solving skills, memories of previous negative experience and uncertainty about the motivation of other people [8].

When starting university many students with AS lack extensive prior social experience – often as a result of having been bullied and left out while at school [9]: '*People have to bear in mind that if you have AS you have probably been bullied for most of your life*' (p.243). Having had limited opportunities to make relationships while growing up the subtleties of communication with strangers can be a bit of a mystery as information is often implied rather than stated. '*Part of the pattern of AS is not being able to read or reciprocate non-verbal communication. In fact, I do not seem to understand it, nor do I fit in well in conventional social situations*' [5, p.4]. Others cannot be relied upon to empathise and may stereotype people with AS rather than appreciating individuality [10].

There is clear evidence that students with AS can acquire strategies which are conducive to success in HE [3,11] however, perfectionism can be a side effect of an imposed identity in school as 'a special needs child'. Sadly under-employment and unemployment after graduation is common, therefore assistance to get onto the next stage may well be needed [12]. Employees with AS can thrive in areas which play to their systematic and logical strengths [4] but the social environment of the work place, and team work, like group work at university can pose difficulties unless handled empathically by colleagues.

To summarise, people with AS may have strengths in systematic logical thinking, application, staying power and hard work, deep narrow interests leading to the acquisition of expert knowledge and perfectionism.

However, challenges may include: other peoples' attitudes and behaviours; imagining novel situations, and working out what to do; utilising effective problem solving, planning and organisational skills, especially in an unfamiliar context; seeing the bigger picture; predicting likely outcomes and understanding other people's motivation, and unclear communication; anxiety about coping (socially, academically and practically) in unpredictable, ambiguous contexts; depression, often relating to feelings of isolation and previous negative peer experiences; issues with the sensory environment; being misinterpreted by others and perfectionism.

A stereotypical mathematics department?

What are typical experiences of mathematics, statistics and operational research (MSOR) study? Analysis of the National Students Survey [13] reports strong positive experiences of students in MSOR. This includes "*criteria used in marking have been clear in advance*" and substantially stronger positive responses than in all other subject areas for "*assessment arrangements and marking have been fair*", "*feedback on my work has been prompt*", "*I have been able to contact staff when I have needed to*", "*changes in the course or teaching have been communicated effectively*" and "*course is well organised and is running smoothly*". This suggests a different and common experience, which might be summarised as a transparent and ordered approach to learning, and is likely to enable students with AS.

In contrast, there are reported negative commonalities – MSOR places near bottom for "*staff have made the subject interesting*". Students with high levels of interest may expect their peers to share this regardless of the teaching. It can be disappointing and confusing if those around you do not seem to have the same passion and this may impact on engagement. MSOR places bottom in "*the course has helped me present myself with confidence*" and "*my communication skills have improved*". These are two areas in which students with AS may find it more difficult to develop their skills independently of their course of study and this could have substantive impacts on accessing placements and employment.

Challis et al. [14] highlight challenges at the transition for all undergraduates in mathematics. It is noted that "*Generally, students are surprised by both the teaching method at university and the nature of university mathematics.*" Challis et al. found that informal support from other students, including those in later years, is particularly important in gaining confidence in new methods and reflecting on them. A student with AS may find the sharp transition to university mathematics particularly challenging and may be less likely to draw on informal peer support.

Attendees at the HE STEM curriculum summit [15] produced a list of intended mathematics graduate attributes that include areas in which students with AS may show considerable strength such as "*the ability (and desire) to learn more and go further*" and "*enthusiasm for the subject*". The list also includes areas which may be more of a challenge such as "*the ability to communicate mathematics*", "*some knowledge of the culture of mathematics*" and "*flexibility*". This latter was highlighted as playing a key role in handling unfamiliar problems.

The Subject Benchmark [16] highlights that MSOR subjects are largely cumulative and hence that ideas can take time to be assimilated. This can be unsettling for all students but may provoke anxiety for a student with AS as attaining understanding may appear unpredictable. A key skill that MSOR graduates are expected to acquire is problem-solving including transferral of expertise to unfamiliar contexts which students with AS may find more challenging. Finally it is noted that seeing arguments developed in real time on the board is an essential part of MSOR teaching. The sensory environment of a lecture may prove distracting for some students with AS and could impact on the ability to process concepts in real time while taking precise notes from the board.

Potential solutions from the academic department

Aspiration raising and early assistance

Pre-entry barriers include lack of encouragement, inadequate information and early support. Beardon and Edmonds [9] participants suggested that staff, including those at open days, should have more awareness of AS. Comments from the Madriaga et al. transcripts [3, p.156] suggest pre-entry information and assistance is helpful and that explicit experience can be encouraging: '*I undertook one week's tasting course [...] it actually proved to work well*'.

In the classroom

Asperger [6] talked about teaching with the affect turned off. Students don't need the emotional overload of another person's frustration. It is easier to understand that which is stated rather than implied. Clarity, predictability, logical explanations and tangible expectations contribute to security and underpin conditions for success. Explicitly communicating that it takes time to assimilate new concepts and teaching study strategies which assist with this may help to reduce anxiety. For students struggling with the sensory environment provision of full notes for annotation can enable the focus to be on understanding.

Outside the classroom

The greatest challenges can occur outside of class. Beardon and Edmonds [9] participants described issues with the sensory environment but noted social and emotional issues more often than anything else. For instance: '*What was the most dreadful was probably what other students looked forward to [...] the breaks between lessons'* (p.157); '*I need a quiet study area*' (p.157); '*Social isolation, depression, anxiety'* (p.158); '*Help understanding social expectations*' (p.165); '*small groups who have similar likes to me [...] places where it's not rowdy*' (p.233).

Academic departments may have a limited role in meeting the social and emotional needs of students but assistance in forming social groups is recommended good practice to support transition for all students in mathematics [14].

A department based work room or allotted times in a Maths and Stats Centre where students are concentrating on completion of mathematics work may provide a place to go between classes, a suitable work environment and ease access into the mathematical community. Clearly timetabled activities within such rooms can provide support to all students. For instance, older peer mentors trained to facilitate informal group problem solving or examining of the 'big picture' may assist all students to form the informal study groups known to be important as well as providing input from older students and practice in communicating mathematics. Structured social elements may also be introduced by departments. For example, the Maths Arcade [17] which brings staff and students together to explore mathematical strategy games and puzzles builds naturally on shared interests and enthusiasms for exploring mathematics and problem solving. Explicit investigation is needed to determine whether such departmental provision might provide a structure which enables students with AS to work in informal groups or socialise within the department.

Academic work

Academic challenges described by Beardon et al. [9] participants relate to organisational difficulties and lack of clarity.

- '*I find it extremely hard to cope with changes*' (p.158)
- '*I felt that I did not understand some of the instructions I was given and just remained silent rather than asking for help*' (p.161)
- '*Tutors not doing what they say they will*' (p.164)
- '*getting lost in detail*' (p.165)
- '*no explanation of how university works*' (p.165)
- '*highly structured learning suits me best*' (p.158)

A dependable, structured, transparent learning environment is one of benefit to all students and given the results reported by Hewson [13] may already be delivered in many mathematics departments.

For some students support from a specialist tutor may be beneficial. A specialist tutor who has sufficient understanding of the transition to HE study, course structure, learning approaches, problem-solving, culture and communication in MSOR can help build skills suitable for mathematical study. Other students may meet regularly with an academic tutor. A shared enthusiasm for particular areas of mathematics can facilitate communication and permit guidance in personal mathematical projects. This may ease frustrations regarding perceived lack of interest from peers or due to limited coverage of areas of special interest in formal lectures as well as increasing confidence and communication skills.

Formal group work is often described as a cause for anxiety, largely around unclear expectations and the feeling that peers are not taking the task seriously. The following is typical.

'*My marks will go down because those bastards didn't do what they said they were going to do and didn't give a toss about whether it was any good or not. This is stressing me out because it has implications for my whole degree classification. I've tried emailing them but they don't bother to even respond and now I am so totally stressed out that I feel like giving up. When we have to present it I am going to look like I haven't even bothered but it's not me it's those other jokers and I really feel like I hate them'*. (Personal communication with Martin, reported with permission).

Beardon et al. [11] suggested that group work could include marks for the extent to which every group member was included, had a clearly defined role, carried it out, and fed back to their team.

Work placement was identified by Beardon and Edmonds [9] as a potential source of agitation mainly around lack of clarity about what was expected coupled often with perfectionism. Academic tutors and departmental placement officers may have a role here alongside staff in Careers and Disability Services. Martin [18] suggests that producing a short introductory statement may work for some students.

REAL services

Some students with AS do not want to access disabled student support because they have chosen to reinvent themselves at university:

'*Having had a diagnosis of Asperger's syndrome in school, as an adult you feel like you can never actually participate normally in everything with everyone else. If you think I am going to go and get any more special needs help, you can get lost.*' (Personal contact of Martin, reported with permission).

Publicising a range of general (rather than disability related) services, and encouraging students to ask for help may be a way forward: '*I find it hard to ask for help [...] almost as if I've failed*' [3]. For instance, provision of support in recognising when and how to access mathematics help is considered general good practice during the transition to mathematical study at higher level [14]. Such provision embedded within the department or a Maths and Statistics Centre, ideally alongside advice on mathematical study skills for all, is likely to be acceptable. Ensuring such services are Reliable, Empathic, Anticipatory and Logical (REAL) has been found to be effective in enabling students with AS [2] and is likely to benefit all students.

Reliability is essential. Students lose confidence quickly if they are let down and the requirement for backup will fluctuate at different stages and transitions. Building a service around a small reliable team is most effective.

Empathy is a two way street. An empathic respect for individuality is necessary in order to avoid the pitfalls of the one size fits all AS support package or the expectation that everyone conform to a stereotype of what students are like. For instance, understanding that group work may be a cause for concern will help staff to work with the student to put something useful in place.

Anticipating that unexpected changes, lack of clarity and ambiguity can cause anxiety is a good starting point. Planning to communicate with clarity, advise in advance of known changes, and to assist the student to get from one stage to another in their course will be helpful. It is quite possible for this to occur with minimum fuss.

University can be a self-esteem enhancing experience as illustrated in the following examples from the Madriaga et al. transcripts [3]: '*I wanted to do the project by myself and I did a very good job actually. It was the first time and I was very proud of it […] I didn't want to go and ask for help because I wanted to show people that I can do something by myself.*'; '*My dream, for as long as I can remember, was to go to university*'.

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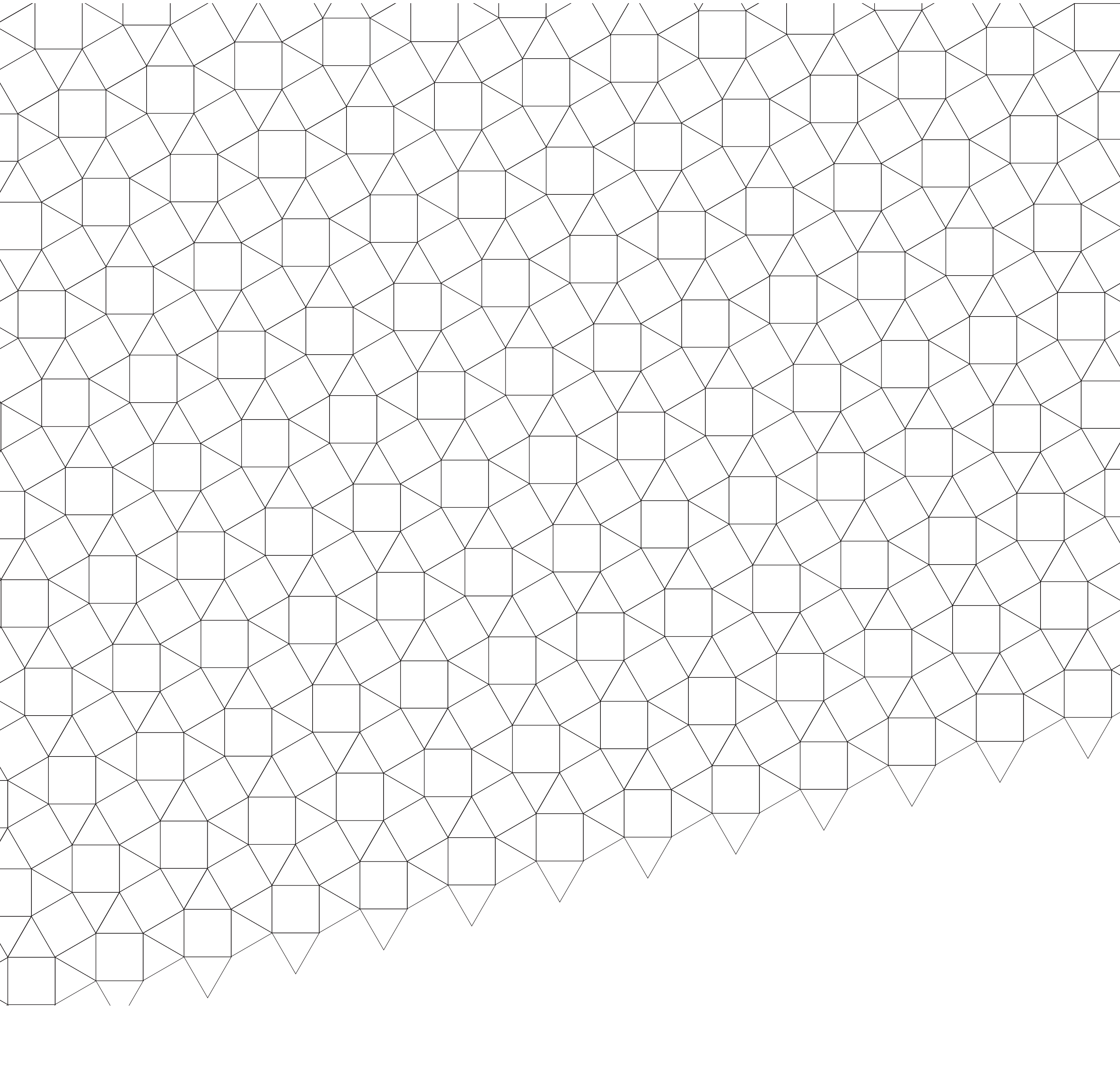

Making online maths accessible to disabled students – issues and lessons from the Open University's experience

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Context

This review of key issues in making maths accessible to disabled students, when presented online, draws on experiences over the last 5 years, or so, at the Open University. The co-operation of colleagues especially: Tim Lowe (Lecturer in Maths) and Jonathan Fine (Technical Developer in Learning and Teaching Solutions) in helping to compile and discuss them is acknowledged with thanks. There is a fuller but older (2008) paper published by the author, et. al. discussing some of these issues, particularly for visually impaired students [1].

Firstly, a few notes about the Open University to set the review in context. The Open University (OU), a world leader in distance learning, attracts many disabled students with its flexible approach to teaching and learning. Here are a few statistics about the OU and comparisons with the averaged statistics for all Higher Education Institutions (HEIs) in the UK collected by the Higher Education Statistical Agency (HESA) [2]. (The OU figures are averages of monthly totals for UK based students for 2010 and the HESA figures are taken from their free online data tables for 2010/11)

Total number of OU active students = 190,000 (9.1% of HESA National Total all HEIs)

Total number of OU disabled students = 11,100 (6.3% of HESA National Total all HEIs)

Percentage of students declaring a disability at OU = 5.8%

HESA percentage of students declaring a disability for all HEIs = 7.3%

Visual Impairment: 11% of OU disabled students; HESA average for all HEIs = 2%

Dyslexia: 24% of OU disabled students; HESA average for all HEIs = 48.9%

In summary, the OU has 6.3% of the UK total of higher education students declaring a disability and the number of its students declaring a disability as a percentage of the total student population is slightly less than the national average. The profile of the disabled student population according to disability is radically different from the average for all institutions. This, as far as is known, has not been studied in depth but it is suspected that key factors here are the preferences of students with different disabilities for different modes of study and support. So, as examples, the OU has many more visually impaired students and much fewer students with dyslexia studying with it than you would expect at most institutions from the nationally average data.

There are two significant distinctions about the OU compared with most other HEIs that are relevant to this review of issues.

1. The OU develops its Modules in teams (sometimes quite large) with a centralised production model for courseware whether printed or now increasingly online.
2. Then as its name suggests the OU is Open Access; there are no academic prerequisites that students (including disabled students) have to have to start study. (There are prerequisites for particular modules and for a student to pass through a programme of study towards a degree). Thus students come to the university with a wide range of prior learning experiences. This presents particular issues that will be discussed later.

Understanding the problem

Web Content Accessibility

This article does not discuss the issues of making web content accessible to disabled people generally. The World Wide Web Consortium (W3C), the open standards body for the Web, has a long established Web Accessibility Initiative (WAI). WAI co-ordinates the production and maintenance of the *de facto* standard for web accessibility: The Web Content Accessibility Guidelines (WCAG 2.0) [3], WAI also produce useful resources that support the uses of these guidelines such as:

- Web Content Accessibility Guidelines (WCAG) Overview [4]
- WCAG 2 at a Glance [5]
- How to Meet WCAG 2.0 [6]

There are numerous other online resources available about web accessibility. To really understand web accessibility the author advocates developing an understanding of how people with diverse disabilities interact with their computer and the content it mediates. This article considers the particular issues when trying to make mathematical content accessible in light of this diversity.

What is different about maths?

In written English it is very largely the linear order of letters that conveys meaning. In maths it is the 2 dimensional relative positioning of symbols, their relative sizes, etc., that codes meaning. Maths is a symbolic language whose representation aids its manipulation. Examples of this are the way we readily cancel identical terms in an algebraic equation, or the mantra (which at least the author learnt at school) for simple differentiation: “bring the power to the front and reduce the power by one” which soon becomes visualized around the symbolic representation.

This facilitation may or may not be preserved in alternative representations intended for disabled people; if not, to what consequence? Because of this symbolic nature maths raises additional issues when presented online in a learning context beyond an alphabetic language.

Interrelated issues across the authoring, technical presentation and teaching and learning processes

How maths is encoded in a web resource is only part of the challenge. There are lots of interrelated issues following from the fact that:

- The maths needs to be authored by lecturers and different tools are available for this
- As well as the standard encoding in the ‘main resource’ in a Virtual Learning Environment (VLE), or other web resource, transformation to alternative formats will be required to meet the needs of some disabled students
- How the maths is presented to the student depends on browser renderings (which vary) and client-side transforms e.g. speech synthesizer with refreshable Braille display
- Students need to be able to interact with the maths not just passively read it; they need to copy it into documents, manipulate it and then communicate it with their tutors or peers
- This student/tutor interaction may have to be facilitated by online communication but in the symbolic language of maths
- The challenges of maths need to be addressed in formative and summative assessments in a way consistent with the student’s experience in their learning

Issues of encoding

Which way of encoding maths in a web page is most accessible to the range of disabled students who will potentially experience challenges with maths online? There is no one simple answer to this question! There are two major options:

- Images with alt-text descriptions
- Specialised maths mark-up – e.g. MathML

Encoding as images

With maths encoded as images (e.g. generated from LaTeX) it is possible to implement the technology so that these images can be enlarged and colours changed; key access techniques for some people with visual impairments and some with dyslexia. For those who cannot see sufficiently well alt texts[1] to such images are a possible solution but for simple maths only. Although alt texts can be transformed to speech or Braille this can be problematic. Creating suitable alt texts is not easy, nor are they easy for the student to interact with for anything but the simplest expressions.

Encoding as MathML

MathML is an XML based mark-up language maintained by the W3C, the open standards community for the World Wide Web [7]. There are two forms of MathML: Presentation Mark-up (how the mathematics should look) and Content Mark-up (which describes the semantics). The standard allows these to be used individually or in combination. Some of the accessibility advances of MathML derive from Presentation Mark-up, some from Content Mark-up. However Presentation Mark-up alone is most commonly used and this restricts the potential accessibility advantages.

Issues of presentation

Where someone is not able to access the standard maths presentation there are two possible tactics:

- Transform the visual representation into a form that can be accessed, or
- Provide an alternative means of accessing the underlying semantics (meaning)

Presentation to the student (or any user) is dependent on: the underlying coding of the maths in the web resource; transformations made server-side (at the University) and how the browser (including plug-ins) interprets that code. Transformations may also be made client-side (by the students' computer and/or assistive technology).

Note on maths and Braille

Braille is important to some learners with visual impairments but only about 15% of people in the UK who are blind (of all ages) are competent Braille users. Some mathematicians who are blind extensively use Braille mathematics code; although a very small percentage of people who are blind ever learn this. There are numerous variations in the way maths is encoded in the Braille schemes of different countries. Firstly English as represented in the Braille codes of the UK and the USA, for example, differs considerably; then UK Braille mathematics code differs from Nemeth Code commonly used in the USA and Marburg used in Germanic countries. Alt-texts of maths expressions rendered as Braille by a screenreader and refreshable Braille display are not expressions in Braille mathematics code but Braille equivalents of the English descriptions of the maths. What should a university's response be to the needs of Braille mathematics code users? If a students preferred way of interaction with maths is through Maths Braille all reasonable steps should be made by the educational establishment to accommodate this. The Open University uses specialist external transcription agencies for this for course texts. However online presentation can be more challenging; e.g. in some formative assessments mathematical expressions are parameterised differently on each visit by the student. It can be difficult to accommodate the needs of Braille mathematics code users in such circumstances depending on implementation.

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[1]An alt text is an attribute within the HTML mark-up of an image. If the appropriate text is entered into it, someone can get a meaningful description of the image via there screenreader and either a speech synthesiser or refreshable Braille display. In this approach the alt txt would be a rendering of the maths in English.

Note on TeX and LaTeX

An example of an encoding of maths, based on print approaches, is the extensive use that has been made of TeX or LaTeX [8]. Many professional mathematicians and authors or editors of texts including mathematics use tools that result in encodings based on these. LaTeX is a document preparation system for the TeX typesetting program. It offers programmable desktop publishing features and extensive facilities for automating most aspects of typesetting and desktop publishing, including important features for maths such as vertical alignment of text. LaTeX, originally written in 1984, has become the dominant method for using TeX and few people write in plain TeX anymore. LaTeX is used as an email exchange format for maths by many mathematicians. Some professional mathematicians who are blind are able to conceptualise directly from LaTeX code and it has become their native format of the symbolic language, i.e. they read, write and mentally process maths in LaTeX. It may be true that students who are blind arriving to study maths at university have also developed this skill in their school education, particularly if they come from a specialist school for students with visual impairments. However this is less likely to be the case for students who need to use mathematics in the context of study of another discipline. So, whereas giving access to LaTeX code may represent a good solution for some disabled students, particularly those with no useable sight, it is not the case for all. This issue is particularly pressing at the OU because students may begin study with limited prior experience of interacting with maths in electronic formats.

Issues of interaction

Maths presented online is not to be just passively read. Students need to interact with and manipulate the maths. They may need to:

- Copy it into documents
- Import it into maths engines (Mathematica, Maple, etc.)
- Exchange maths expressions with tutors and peers in forums, emails, etc.

So the tools they use for this and their compatibility with the online coding needs to be considered too.

Issues of pedagogy

Mathematics is used differently in different contexts. For example compare a Level 3 Maths course (equivalent to final year of an undergraduate course) with the basic calculations in an introduction to social-science course or more markedly, an Openings course[3]. What learning path are we a seeking to take the student through over their period of study? For the disabled student this might include a transition in the way they interact with mathematics. It should be noted that choices made about how maths is presented online do not just impact on the possibility and ease of perception of the notation but on the learning itself.

It is important not to forget assessment. Basically the techniques and accommodations students use in their learning must be available in their assessments both summative and formative. However a full discussion of access issues related to maths in assessment is beyond the scope of this brief report.

Current practice at the OU

The maths department normally writes courses in TeX; however other departments and faculties vary and common practice includes the use the Equation Editor for Word. For long texts PDFs are produced with figure descriptions. Audio descriptions may be recorded. This is the case for MU123 (a pre-calculus basic level maths course with approximately 3,500 students registering each year). The OU's VLE program (2005-2008) was a major investment by the University in

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[3]The Openings courses are for those who are new to Higher Education or those who have not studied for some time and may not feel confident about their study skills. An Openings course gives the student a gentle introduction into higher education.

adopting and adapting Moodle[4] integrated with a range of other technologies and tools. One of the sub-projects of this programme produced a MathML filter. If MathML is present in a web resource it is processed, server-side, so that it can be displayed irrespective of which browser or associated plug-ins the student is using. Students can select user options so that either a graphics presentation is served or access to the MathML code is made available to their browser and any assistive technology. The default setting is for the display of images. VLE user settings cover size and colour contrast preferences for those students who need to adjust these to readily read maths. However these approaches are still not used a great deal by students; but have also not been widely promoted. There has only been 1 presentation of introductory course MU123 so far. We don't really know how helpful these approaches are being to disabled students. Some non-disabled students are using image enlargement (picked up from forums) where others have discovered that TeX within $$ … $$ is rendered by Moodle (this is a Moodle feature not turned off – but not advertised).

Browser issues with MathML – issues of support for the OU

There can be problems for a student setting up for MathML (different browser issues). IE requires a free plug-in from Design Science[5] called MathPlayer (they have recently developed the plug-in for I.E. 9.0). Firefox does not support MathML in HTML but does support it in XHTML; the Chrome, Safari and Opera web browsers support MathML natively (however there are some version and consistency of display issues). This presents an issue for the users (students) and an IT support challenge so, as stated above, the OU-VLE defaults to displaying images.

What the OU is seeking to move towards

Two high level objectives are to get maths content on the VLE in a more interactive fashion and to more easily manage student choices of what presentations of maths best suits them. There are some known usability issues we want to address. For example: currently, when in a quiz, where radio buttons are displayed next to an expression, rendered from MathML, it is easy to click on the expression instead of selecting the radio button, this is due to a Design Science plug-in feature, to magnify on hover, having unintended consequences. We also want to overcome a current problem where users experience some delay when loading web pages with multiple images of maths expressions.

Outstanding issues

The fact that there is no one right answer for how maths should be handled in web content means different people within the University argue for different solutions to key issues. Given the OU's centralised production model, one issue of particular importance is: what should be specified as the base format that maths should be stored in; before transformation to other formats as required? Some argue for MathML others TeX. Given that selection then what are the best authoring tools to offer academics that need to write courses containing maths? MathML is currently only used on a few courses, so we have had limited experience with it.

A fundamental issue is that we don't know enough about the users (the students) and their experience of interacting with maths online. This is a planned subject for future research. A new course M347 Mathematical Statistics will be making substantial use of MathML. It is the first third level maths course to be presented on the VLE only. Its first presentation is due in February 2012. This will be a crucial course for institutional learning; we will know a lot more after we have gone through the development and first presentation of this course.

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[4]Moodle is a popular open-source Course Management System see: http://moodle.org/ (Accessed 31 August 2011).

[5]Design Science is the leading company in making maths accessible online through MathML based approaches. They have long been involved in developing free plug-ins that enable Internet Explorer to display MathML and produce a range of authoring tools for maths in web content and Word and DAISY electronic documents. The company's website is available at: http://www.dessci.com/en/ (Accessed 31 August 2011).

Unresolved dilemmas

There are sometimes issues of how to meet disabled students needs without reducing quality for other students. An example of this occurs in current situations where MathML is used. It is estimated more than 95% of students are not receiving any benefits where MathML is being used. Arguably they receive a poorer visual rendering of the maths than they would have with images generated from LaTeX. However it should be noted that no complaints about this have been received. There is the future possibility of supplementing MathML with images supplied from original TeX to address this.

There are curriculum issues, of 'graduateness'. Should not all maths graduates be able to produce nicely formatted printed maths, therefore should we teach TeX/LaTeX? Furthermore should mathematical notation itself be part of the curriculum? However this is probably not an issue for the numerous other disciplines that use maths.

Concluding comment

Making mathematics fully accessible to a diversity of people in online learning is not a solved problem. Nor are the options for presenting maths online generally optimally resolved. However it is clear that one solution will not suit all users and contexts and thus flexibility is key.

Recommendations

When considering the best approach to managing maths presented online, so that it is accessible to disabled people, the problem needs to be considered in the round and include:

- How the students (disabled and not) interact with the maths in the context of their learning
- How academics are enabled to author the maths for online presentation
- How transformations to the different formats required by disabled students are to be achieved in an effective and efficient way

At the OU we have found we require approaches based both on MathML and TeX/LaTeX and this is likely to remain the case for the foreseeable future. This will probably be the case in many HE contexts. So the principle recommendation is to review how best to integrate both these approaches, to best meet the needs of all stakeholders. It is hoped that this article has shared something from the experience at the OU that helps other HEIs formulate what is the best approach in their context.

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[7] See: W3C Math Home: http://www.w3.org/Math/ (Accessed 31 August 2011)

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MathML and speech text

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Context

Use of a screen reader often helps students with visual impairments, and also those who have problems with symbolic notation. For this to work for mathematical content, the content must be translated to speech text, which can then be read by a screen reader.

This case study is based on the author's evaluation of some course materials produced at the Open University, his specification and programming of a transformation of Presentation MathML[1] to speech text [1], and conversations on the Blindmath mailing list [2].

The views expressed here are the author's and not necessarily those of the Open University.

Potential barriers

There are several problems in the use of MathML to provide audio rendering of mathematical content. They can be divided into four groups.

1. The medium does not support MathML.
2. The medium does not support translation of MathML to speech text or audio.
3. The MathML uses inappropriate markup.
4. The translator to speech text produces inappropriate output.

Media support for MathML

Web pages are the only mainstream media that provide good support for MathML, and then sometimes only with difficulty. For example, much blog and forum software does not support MathML, and neither does email.

Media support for translation of MathML to speech text

So far as I know, the Internet Explorer web browser together with the freely available MathPlayer plugin from DesignScience is the only combination that supports translation of MathML to speech text, for use by a screen reader.

Alternatively, the MathFlow tool (which has to be purchased from DesignScience) provides translation of MathML to speech text as a Windows command-line program.

MathML uses inappropriate markup

Even though an equation in MathML looks right, it might not sound right. I have seen examples of this produced by MathType, and also by TeX to MathML conversion software.

- Broken numbers. A decimal number, such as 12.34, might be read as two whole numbers, such as '12 34'. This will happen if, for example, the MathML is marked up as <mn>12</mn><mo>.</mo><mn>13</mn>. I have seen this problem in MathType equations, where it may be due to incorrect authoring.

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[3]Please refer to Cooper pp. 39-45 of this collection for an explanation of MathML.

- MathML using the <mrow> element for grouping, much as LaTeX uses curly braces '{' and '}' for grouping. In MathML unnecessary grouping does not change the appearance of the MathML but can affect its translation to speech, by introducing breaks within numbers (as above) or extra pauses between words.

These problems can also arise when spaces are placed in a number, to act as a thousands separator.

The translator produces inappropriate output

First, here is a confusing example of apparently incorrect output. Given a lambda character as input, MathFlow (and presumably MathPlayer) produces 'lamda' as speech text. The JAWS screen reader speaks 'lambda' incorrectly. It voices the 'b'. However, JAWS says for 'lamda' what it should say for 'lambda'.

According to Wikipedia [3], there are five Unicode characters that can be used for 'approximately equal to'. One of them, U+2245 (which is an equal sign with tilde on top) is often used to mean 'is isomorphic to'.

Equations such as 'A = B = C', where A, B and C are large expressions, are often formatted for display as a table with two columns. MathML to speech text conversion, not knowing any better, reads the table structure as well as the equation. Matrices have the same problem, and when the entries are numbers the signal-to-noise ratio is quite low.

Similarly, a cases statement when formatted for display is a table, and the translator will read it as a table rather than a cases statement.

When text, such as 'the price of bread', appears in MathML it is often formatted with non-breaking spaces as spaces (perhaps to get it to work in the web browser). However, MathPlayer ignores the spaces, producing 'thepriceofbread'.

Conclusions

In a useful but limited range of circumstances, mainly web pages, MathML can be used to make mathematics accessible to the visually impaired. Sometimes MathML can look right but not sound right, because the translator does not read the visual representation.

Recommendations

These are recommendations for making mathematics accessible to screen readers. They may be inconsistent with other accessibility recommendations, such as the use of talking book software.

A. Use a medium that supports MathML (which in practice means web pages).

B. Use software that supports a MathML to speech translator (which in practice means Internet Explorer on Windows and MathPlayer).

C. Test that the speech text is correct (a visual check is not enough).

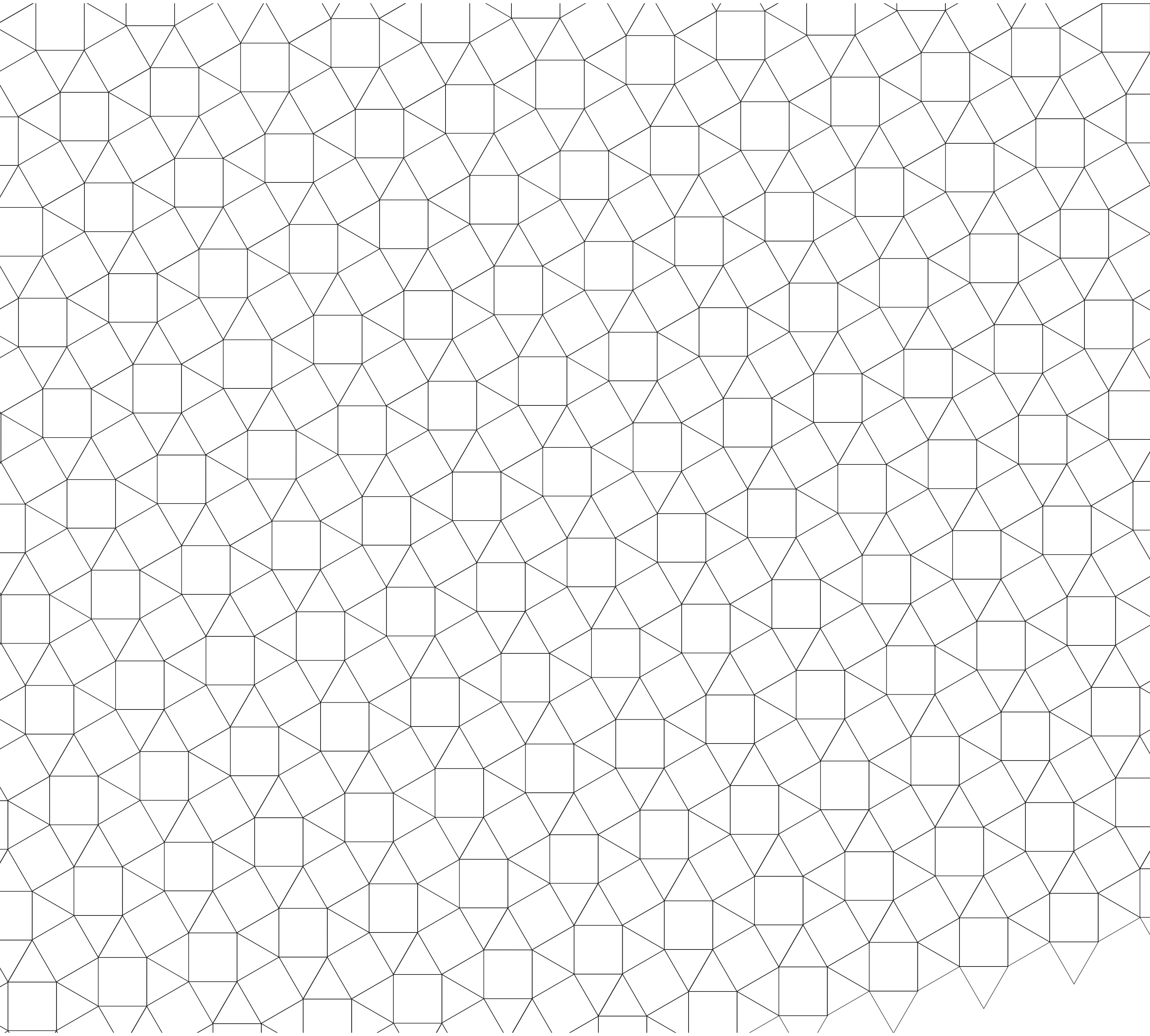
D. Be aware of the problems introduced by implicit tables.

References

[1] https://bitbucket.org/jfine/ou-lts-mathml-to-speech-text/wiki/Home

[2] http://www.nfbnet.org/mailman/listinfo/blindmath_nfbnet.org

[3] http://en.wikipedia.org/wiki/Equals_sign#Approximately_equal



Use of mindmaps to improve accessibility of the mathematics

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Introduction

The aim of this article is to reflect on and promote discussion of accessibility of teaching university Mathematics. There are a number of features of 'traditional' Mathematics teaching that can unintentionally inhibit the participation of some disabled students. I will suggest a possible way to remove these barriers through the use of mindmaps and teaching technologies.

Context and barriers

Long sections of text seem to be the cause of problems for many students with Asperger syndrome (AS) and dyslexia. Students with AS report that they can focus better if the material is broken down, otherwise they find that they can got lost half way through, lose their place or fail to retain the required information [1]. Similarly for students with dyslexia who report problems when reading mathematics when it is embedded in large amounts of text [2]. Multistage problems in Mathematics can produce a similar effect, with the end result typically being that the student only focuses on one part and fails to answer the question as a whole [3]. Mindmaps (also called tree diagrams) have been shown to help. Trott describes an example of where a student with dyslexia had difficulty with partial differentiation. She describes in [2] how the student got lost and often omitted derivatives. They tried a mindmap and this helped the student identify the various aspects of the problem as well as ways in which they interplay. This imposed an organization to the student's work which was previously lacking. Mindmapping allows the user to view information in a way that is selective and make links between the information to give the bigger picture. Mindmaps consist of a series of branches which can contain a specific part of the methodology. Branches can collapse at a glance to hide information so that the user can focus on a particular part and the user can view one branch at a time, without having to sift through large amounts of text.

Numerous lengthy, multistage problems and large sections of text is a current feature of the Mathematics courses that I teach and, for reasons outlined in [4], these may pose as unintentional barriers to some disabled students.

Recommendations

Mindmapping may be a way of improving accessibility and incorporating a degree of flexibility that will likely benefit all students. Figure 1 (overleaf) illustrates an example of how a mindmap diagram can help organise an extended piece of mathematics.

The work described in Figure 1 is a summary of the eigenvalue/eigenfunction method for solving two-dimensional linear systems with complex eigenvalues. Even the summary of this process is rather lengthy (Fig 1a, top panel). The mindmap equivalent (Fig 1b, bottom panel) provides a clear structure without the distractions of the text, encouraging the student to focus on the key steps before sinking into any detail. The branches of the mindmap also work well as 'memory hooks' and thus particularly important for students with poor short-term memory.

One possible way to construct these mindmaps is to use the software package MindGenius. MindGenius combines technology and mindmapping. It is flexible, in that any number of colours, text fonts and sizes can be used, and it has the ability to fit into any environment, i.e. exported directly into any number of formats e.g. Word, Powerpoint, PDF, HTML without any

Summary: complex eigenvalues

- **Case 1:** A is a 2-by-2 matrix:

Step 1: Find eigenvalue $r = \lambda + i\mu$ and its complex conjugate $\overline{r}$.

Step 2: Find the corresponding eigenvector ξ_r for r: i.e. solve $(A - rI)\xi_r = 0$ to find $\xi_r = (\xi_1, \xi_2)$.

Step 3: Find the $x^{(1)}(t)$ soln (by Euler formula):

$$x^{(1)}(t) = \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} e^{\lambda t}(\cos(\mu t) + i \sin(\mu t))$$

Step 4: The 'fundamental set' is then:

$$u(t) = \mathrm{Re}\, x^{(1)}(t), \quad v(t) = \mathrm{Im}\, x^{(1)}(t).$$

Step 5: The 'general soln' is

$$x(t) = c_1 u(t) + c_2 v(t).$$

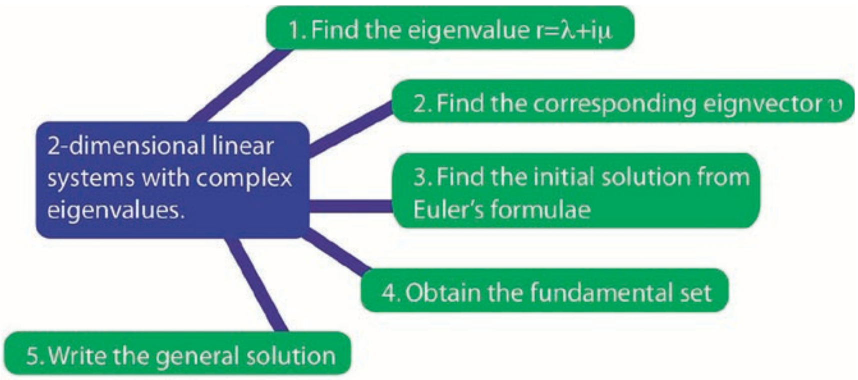


Figure 1: (a, top) A summary of the eigenvalue/ eigenfunction method for solving two-dimensional linear systems with complex eigenvalues. (b, bottom) A mindmap of the same procedure.

additional re-typing. MindGenius can be used effectively for planning, organising notes and studying or remembering information. It fits nicely with the needs of some dyslexic students, who report to prefer to learn from a visual source [2], as well as students with AS, who may find it difficult to structure their notes for revision purposes [1]. Additionally, it is possible to attach sound (MP3) files to branches of the map, which may be of great benefit for, for example, a person with dyslexia or visual impairment to have lecture notes in an aural content.

Another key advantage of using mindmapping is that it facilitates a nonlinear presentation of the material. The traditional teaching approach, along with the typical modularised course structure, goes a long way in forcing a linear way of thinking in our students [5] and this appears to be detrimental to many students, particular those with short term memory problems, such as those with dyslexia and AS [3]. In my experience, students generally struggle to apply prerequisite material and relate it to new course material. In many cases, students lose interest or are unable to engage when new material is taught that relies on previously taught techniques. One of my students who is diagnosed with AS once told me that the material that I was teaching was foreign to him, even though the student was simply being asked to apply previously taught material, which I knew he had a strong grasp of, to a new problem. The solution, in this instance, was to reteach the material to the student in the framework of the new problem. Obviously, there is simply not enough time to reteach everything and, even if there was, it would not be beneficial to all students and could easily cloud the focus of what you are actually trying to teach, i.e. the new material. However, I have found a possible solution in the form of 'layers' in the mindmap. The idea is that, with a single click, the student can, if they need to, uncover the necessary prerequisite material. For example, in Figure 1b, step-one in solving a

2-dimensional linear system is to find eigenvalues of a 2 by 2 matrix. This was taught to the students in a previous year and typically I would just ask them to refer to that material. Now, I just tell them to click box 1 to reveal the necessary method (as shown in Figure 2).

Eigenvalues of a 2x2 matrix

1. Consider $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$.
2. Solve the determinant $|A - \lambda I| = 0$ to give $\lambda^2 - (a+d)\lambda + ad - bc = 0$.
3. The eigenvalues of A are then the roots of this quadratic, namely

$$\lambda = \frac{a+d}{2} \pm \frac{1}{2}\sqrt{(a+d)^2 - 4(ad-bc)}.$$

Figure 2: the method for finding eigenvalues of a 2 by 2 matrix. This information is revealed to the student when they click on the mindmap box 1 in Figure 1b.

In a similar way, the student can, if needs be, remind themselves of the definition of a ‘determinant’ by clicking the highlighted text. This way the student can see the previously learnt material in the context of the current problem and then, hopefully, it would appear less foreign. The application of mindmaps in all of my teaching is still work in progress but there are clear advantages in that they help signpost the key points, provide easy access to prerequisite material, are, importantly, transferable to a number of accessible formats which help remove some of the barriers placed by the traditional university mathematics teaching approach [4].

References

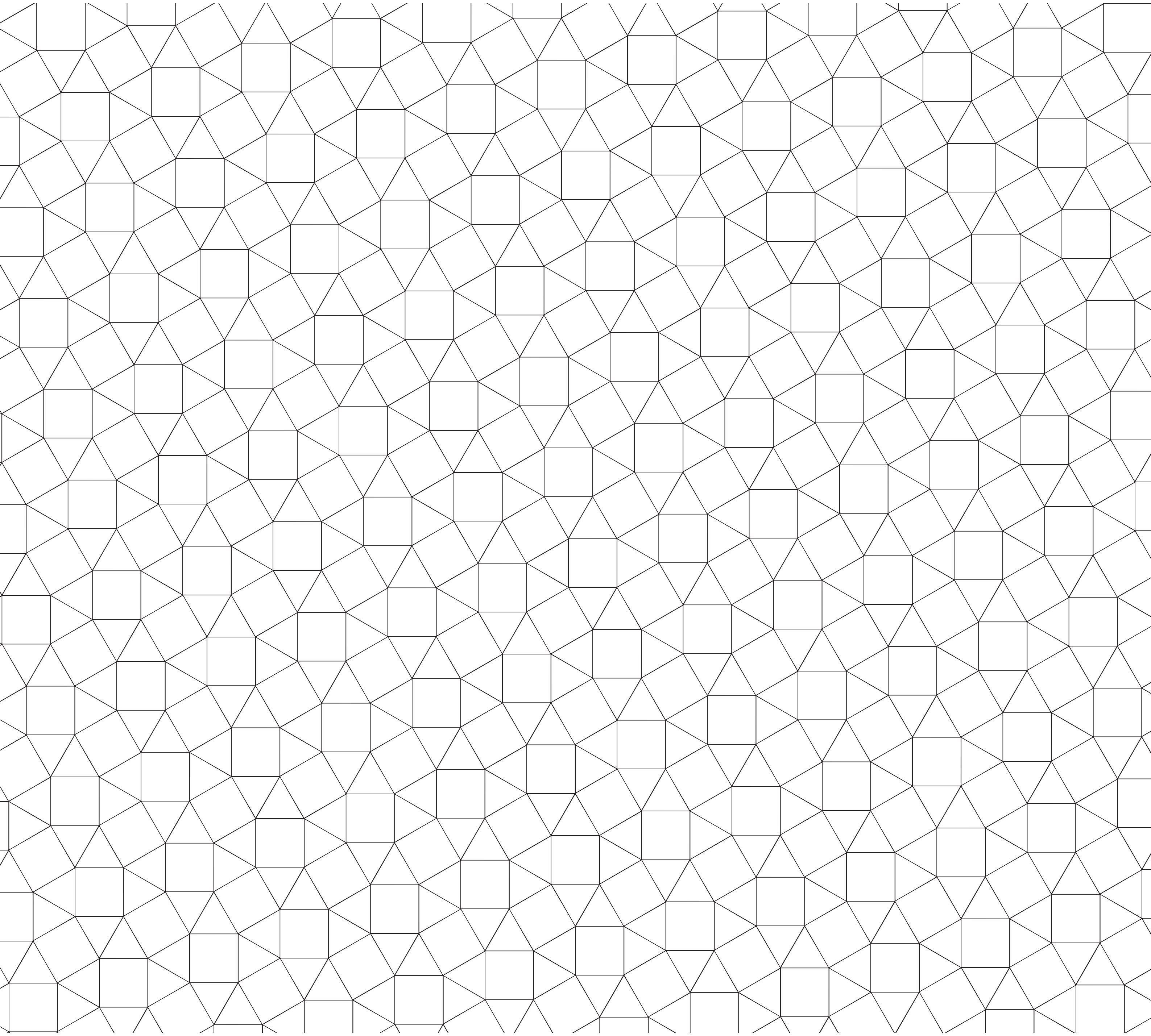
[1] Hughes, M. (2008) *Higher Education Academy event on Asperger’s syndrome*, 3rd Dec. Strathclyde University.

[2] Trott, C. (2003) Mathematics support for dyslexic students. *MSOR Connections*, Vol. 3 (No. 4):17-20.

[3] Rowlett, E.J. (2008) Accessibility in MSOR: one student’s personal experience. *MSOR Connections*, Vol. 8 (No. 1):27-30.

[4] Webb, S.D. (2011) Accessibility of University Mathematics, *MSOR Connections*, Vol. 11 (No. 1):42-45.

[5] Kinchin, I.M., Hay, D.B. (2000) How a qualitative approach to concept map analysis can be used to aid learning by illustrating patterns of conceptual development. *Ed. Res*, Vol. 42 (No. 1):43-57.



Selected further information and resources

This guide concludes with a collection of references to resources, sources of further information and papers. This list is by no means exhaustive, highlighting only some key sources. We hope that by using the list as a starting point departments seeking MSOR specialist information will be able to discover resources more effectively.

Guidance on the web

- **W3C. W3C Math Home.**
  http://www.w3.org/Math/
  The W3C Math Home is a central point of access for MathML specifications, resources, software, tutorials, references, articles, links and news. The software list includes accessibility, converter, editor and browser subsections.
- **Formats and Guidance. UK Association for Accessible Formats.**
  http://www.ukaaf.org/formats-and-guidance
  This UKAAF site collects together documents and links to publically available resources on the creation of accessible formats. This is general information and does not focus on mathematics or scientific documents. However, some linked sites do contain subsections on mathematics and those on audio transcribing and tactile graphics are generally useful.
- **DAISY Consortium. MathML.**
  http://www.daisy.org/project/mathml/
  DAISY is the international Digital Accessible Information System standard which allows the production of flexible e-books mixing text and audio from audio-only, to full text and audio, to text-only. The DAISY Consortium MathML pages link to the MathML in DAISY specification, sample DAISY books which contain mathematics and a very useful list of general resources for accessing mathematics which includes mathematics-enabled DAISY readers.
- **InftyReader Group. Flowchart of our Process for Developing Accessible Math Materials.**
  http://www.inftyreader.org/flowchart.htm
  This diagram provides a quick insight into the current process to produce accessible mathematical documents when using a small range of commercial assistive technology specifically developed for scientific documents containing equations. Standard assistive technology will usually not suffice.
- **Design Science. Solutions for the Accessibility Community.**
  http://www.dessci.com/en/solutions/access/
  Design Science has delivered a number of accessibility products for mathematics which are linked from their access pages. They also provide a listing of which mainstream accessibility products (e.g. screenreaders) provide mathematics support. Also linked from their main access page is a definition of, and short introduction to the importance of mathematics accessibility and pages which highlight the technology issues.
- **Department of Physics, Oregon State University. Science Access Project (SAP).**
  http://dots.physics.orst.edu/
  This is now rather old (last updated in 2005) but tracks the origins of DotsPlus and ViewPlus. Informative US pages on Braille, DotsPlus and a list of publications (1993 – 2004) which document the work on the DotsPlus extension to the Braille code, the Tiger Tactile Graphics and Braille Embosser technology and audio graphing. ViewPlus Technologies (founded 1996) commercialised this work but the SAP site provides useful background.

- **Access2Science. Topics in STEM Accessibility.**
  http://www.access2science.com/index.html
  This site was started in 2011 to bring together practical information, articles and links on the accessibility of STEM subjects. At the time of writing the collection is relatively modest but it is hoped this will become a useful resource. The editors are *'all volunteers, are blind, have earned PhDs, and are executives in companies or agencies that market information accessibility products or services.'*
- **Texas School for the Blind and Visually Impaired. Math Home Page.**
  http://www.tsbvi.edu/math-home-page/
  These pages are compiled/written by Susan Osterhaus a secondary level mathematics teacher at the Texas School for the Blind and Visually Impaired. They provide a guide to teaching at this level although some information is specific to Nemeth code. However, pages on teaching strategies, project Math Access, Tactile Math Graphics, accessible maths tools and resources are practical and useful.
- **Kapperman, Gaylen, and Jodi Sticken. Project Math Access.**
  http://s22318.tsbvi.edu/mathproject/
  This website was developed by the staff at the Research and Development Institute of Sycamore, Illinois. It collects together practical information on approaching the teaching of mathematics, including advanced topics, to students who are blind. In addition the site hosts an online version of the majority of *The Handbook for Spoken Mathematics* (Lawrence Chang, 1983). The section on Diagrams and Graphs is not included.
- **Mathspeak and teaching mathematics. Abraham Nemeth.**
  http://people.rit.edu/easi/easisem/talkmath.htm
  http://people.rit.edu/easi/easisem/nemeth1.htm
  Prof. Nemeth is Professor Emeritus of Mathematics at the University of Detroit Mercy. He has been blind since birth. He developed the American Braille mathematics code, Nemeth Braille Code for Mathematics and Science Notation and MathSpeak, a system for orally communicating mathematical text. In these two articles Prof. Nemeth describes how he trains his readers and how he approaches teaching.
- **Dietrich, Gaeir, John Gardner, Neil Soiffer, and Steve Jacobs. Resource List for Accessing Math and Science.**
  http://accessiblemath.org/resources.htm
  This is a short and to the point list of resources for accessing maths and science produced by individuals representing the High Tech Center Training Unit of the California Community Colleges, ViewPlus Technologies, Design Science and the InftyReader Group for the 25th *CSUN Conference*.
- **Maths, Stats & OR Network. Supporting Students with Disabilities.**
  http://www.mathstore.ac.uk/node/126
  This page collects links to material published by the Maths, Stats & OR Network on supporting students with disabilities. Included are links to all articles published in *MSOR Connections* on this topic, *MSOR* funded projects in this area and a small collection of links to other sites of interest.
- **SCIPS. Mathematics, Statistics and Operational Research.**
  http://www.scips.worc.ac.uk/subjects/maths_research.html
  SCIPS is a resource developed by Val Chapman, University of Worcester, offering strategies for inclusive teaching and learning at degree level. The pages for mathematics, statistics and operational research provide an overview and links to the subject benchmark, groups of learning activities on such degrees and potential barriers, case studies and external resources.
- **Methods and materials for teaching science to deaf students.**
  http://ideatools.rit.edu/hgl9008/msse/
  Produced by Rochester Institute of Technology these are resources for their *Master*

of Science in Secondary Education. These extensive resources include teaching and communication strategies, curriculum development, technologies and a math/science specific bibliography. Of less use are those resources making specific reference to American Sign Language (ASL) as this is not mutually intelligible with British Sign Language (BSL).

- **ScienceSigns, EngineeringSigns and Maths and Science Glossaries.**
  http://www.sciencesigns.ac.uk/
  http://www.engineeringsigns.ac.uk/
  http://www.ssc.education.ed.ac.uk/bsl/list.html
  It can be difficult for students who are d/Deaf, Communication Support Workers and Interpreters to translate specialist terminology in science, engineering and mathematics. The above sites collect together such terminology and also contain key information on how these collections were produced. There is no single collection of signs for higher level mathematics but certain facets of the terminology/vocabulary are available on the above sites. A student working with a non-specialist interpreter or communication support worker may appreciate you highlighting the availability of these collections.
- **Technology For Supporting Dyslexic Students Using Mathematical Notation And Scientific Language.**
  http://www.dyslexic.com/articlecontent.asp?CAT=Technology&slug=249&title=Technology%20For%20Supporting%20Dyslexic%20Students%20Using%20Mathematical%20Notation%20And%20Scientific%20Language
  This article provides a brief overview of technology which dyslexic students may find to be of assistance. This page notes some of the difficulties using some 'standard' support technology in mathematics and the sciences and offers guidance on technology which is likely to be more successful.

Forums and mailing lists

- **Blindmath. Blind Math list.**
  http://www.nfbnet.org/mailman/listinfo/blindmath_nfbnet.org
  This is a list based in the United States provided by the National Federation of the Blind (NFB) and dedicated to the discussion of any issues related to blindness and mathematics. Anyone with an interest in this area is free to join and contribute to discussions.
- **AccessMSORWG. JISCMail - ACCESSMSORWG List.**
  https://www.jiscmail.ac.uk/accessmsorwg
  This list is intended to provide a forum for members of UK HE institutions with expertise or interest in accessibility and issues surrounding supporting disabled students in mathematics, statistics or operational research (MSOR) subjects. Questions on aspects of MSOR access are welcomed and members are encouraged to share good practice, advances in technology and relevant news, workshop information or calls for conferences/participation.
- **The Dyslexia and Dyscalculia Interest Group (DDIG).**
  http://ddig.lboro.ac.uk/
  The DDIG project (2003-2005) brought together higher education practitioners from mathematics support centres and additional learning support to exchange information and raise awareness of the links between dyslexia, dyscalculia and the study of mathematics in higher education. The DDIG website is the legacy of this project and brings together useful resources in this area. The DDIG mailing list, linked from the main page remains as a bulletin service.

Assistive technology and software for mathematics

- **How can Design Science products make math accessible? Design Science.**
  http://www.dessci.com/en/solutions/access/accessibilityfeatures.htm
  MathType is an equation editor which can be used either with mouse or keyboard and which

has some attributes which recommend it to users who are partially sighted or who require a large on-screen target for mouse clicks. MathType can also be used when authoring accessible resources by individuals. MathPlayer enhances Internet Explorer to display, enlarge and speak mathematical expressions authored in MathML. Some screenreaders and text to speech engines work with MathPlayer.

- **Accessible Pages with MathJax. Neil Soiffer.**
  http://www.mathjax.org/resources/articles-and-presentations/accessible-pages-with-mathjax/
  MathJax allows display of MathML or LaTeX in web pages in all browsers. Equations can be scaled with the page or zoomed. If MathPlayer is present and Internet Explorer is used then MathJax works with MathPlayer to make the mathematics accessible to screenreaders, magnifiers and literacy support software. This enables MathPlayer to access equations written in LaTeX not just MathML.
- **About sAccessNet. Science Accessibility Net.**
  http://www.sciaccess.net/en/about.html and the Infty Group http://www.inftyreader.org/
  Science Access Net produce InftyReader, Optical Character Recognition for scientific documents (good quality scans and PDF) including equations. Output can be corrected in the freely available editor InftyEditor and output in LaTeX, Human-readable LaTeX, MathML, Word 2007 and IML formats. IML is the internal format which can also be opened in the voice enabled editor ChattyInfty. ChattyInfty allows direct control with voice interface of InftyReader, input and editing of scientific documents with voice interface and LaTeX output to a refreshable Braille display. InftyEditor not only provides a correction interface for support workers but can be used as a note-taking interface where IML output is required. It also provides a simple pen interface for limited handwriting recognition of mathematics.
- **Math Access. Viewplus.**
  http://www.viewplus.eu/solutions/math-access/
  Viewplus provides Tiger Braille Math which translates Word documents containing maths to Nemeth, LaTeX, UK and French mathematics Braille codes. Braille can only be printed on a Tiger Embosser. The IVEO system provides a combined touch, sound and sight interface for mathematics. AGC is a scientific calculator with audio and tactile feedback.
- **MathTrax, NASA.**
  http://prime.jsc.nasa.gov/mathtrax/
  MathTrax is a graphing interface with audio feedback and graph descriptions which works with screenreaders. It is freely available and can be used to graph equations and work with datasets.
- **Zyfuse. Zychem Ltd.**
  http://www.zychem-ltd.co.uk
  The Zyfuse heater can be used to produce tactile diagrams. Black ink (pen or certain photocopiers) is applied to Zy-Tex2 paper. When the paper is passed through the heater the black ink swells to create raised lines/areas. This method can be used for in house production of tactile diagrams from simplified diagrammatic materials.
- **Lambda Project. Guiseppe Nicotra et al**
  http://www.lambdaproject.org/default.asp?sec=1
  The Linear Access to Mathematic for Braille Device and Audio-synthesis (LAMBDA) project was a project funded by the European Commission and involved 14 partners in 7 countries. The first main output is the Lambda code derived from MathML but designed to be used with Braille and vocal synthesis and can be converted to MathML, LaTeX, MathType etc. The second output is an accessible editor system using the code aimed at secondary school to university level which can be used to write and manipulate mathematical expressions in a linear way.
- **LaTeX-access. Alastair Irving, Robin Williams.**
  http://latex-access.sourceforge.net
  The LaTeX-access scripts were created by two mathematics students in the UK and can be

used to provide real-time translation of LaTeX into Nemeth Braille and English speech. They are freely available and aimed at those who wish to read LaTeX documents on a refreshable Braille display with or without speech. The scripts do not translate whole documents but concentrate on on-the-fly translation.

- **BrlTex. Michael Whapples.**
  http://brltex.sourceforge.net/
  BrlTex is a freely available LaTeX to Braille translator written by a former physics student in the UK which can be used to translate whole documents. It is designed to handle multiple maths codes (see site for current status). Development of BrlTeX was support by the MSOR: LaTeX and Braille project.
- **LaTeXLex. Mesar Hameed, Emma Cliffe.**
  http://mesarhameed.info/projects/latexlex
  LaTeXLex is freely available software written by a computer science student in the UK. It can be used to clean up whole LaTeX documents into a human readable LaTeX to increase readability of the content in speech and Braille. Alphabets such as Greek can optionally be replaced by UTF-8 symbols, if these can be read, to shorten the content further.
- **What about braille math? Duxbury Systems.**
  http://www.duxburysystems.com/faq2.asp?faq=15
  http://www.mackichan.com/index.html?products/snb.html~mainFrame
  Duxbury Systems produces two Braille translation programs which can translate mathematics to Nemeth code and in the case of DBT, the codes of some other countries. DBT works with some LaTeX files, specifically Scientific Notebook produces LaTeX files which DBT can read.
- **MathTalk. Metroplex Voice Computing, Inc.**
  http://www.mathtalk.com/products.htm
  Metroplex provides speech recognition for mathematics by working with Dragon NaturallySpeaking and either MathPad or Scientific Notebook. MathPad is a basic version allowing input of spoken arithmetic while MathTalk with Scientific Notebook can input a very large set of commands and is suitable for degree level. It should be noted that the latter includes MuPad and graphing and so is unlikely to be appropriate in an examination setting.
- **TalkMaths, Kingston University London.**
  http://talkmaths.sourceforge.net/about.php
  TalkMaths is freely available speech recognition for mathematics under development at Kingston University. This is a prototype system which works with Dragon NaturallySpeaking 9 and allows the user to speak mathematical expressions into MathML or LaTeX.
- **LyX.**
  http://www.lyx.org/
  LyX is a freely available WYSIWYG editor for LaTeX. It provides a graphical interface or keyboard input for mathematics. While not an assistive technology it can be of assistance to students who need to use a computer to write or do mathematics and who benefit from auto-completion, immediate visual rendering, copy and paste from LaTeX code, spellchecking, thesaurus, revision tracking, an outline mode and location of source from viewer. It can assist students learning to type mathematics and who will later go on to use LaTeX but need scaffolding to learn the commands. Documents can be output in MathML and hence can be read back using other assistive tools.
- **EmPower program. Efofex.**
  http://www.efofex.com/empower.php
  Efofex provides input environments for equations, mathematical diagrams, graphs and statistics at a school level. It is sometimes provided to school students who have difficulty producing handwritten mathematics and so students may have used this prior to studying at degree level. The input environments differ from others mentioned here so a student may

continue to use this at degree level alongside learning a tool which is suitable for higher level mathematics.

- **Dasher (with LaTeX).**
  http://www.inference.phy.cam.ac.uk/dasher/
  http://www.inference.phy.cam.ac.uk/dasher/presentations/Closing05/mgp00082.html
  http://www.inference.phy.cam.ac.uk/dasher/Languages.html
  Dasher is a freely available input mechanism which can be used with a wide variety of non-keyboard input methods (mouse, trackpad, touchscreen, rollerball, joystick, foot mouse, head mouse, gazetracker). Experienced users can achieve input speeds of 20-30 words per minute with a gazetracker and 29-39 words per minute with a mouse equivalent. The interface uses zooming and a language model to predict more probable zoom areas. A LaTeX alphabet and training file are available so Dasher can be set up to write LaTeX source code for mathematics input.

- **Math Input Panel. Windows 7.**
  http://windows.microsoft.com/en-GB/windows7/Use-Math-Input-Panel-to-write-and-correct-math-equations
  http://office.microsoft.com/en-us/onenote-help/using-onenote-with-a-tablet-pc-HA001192415.aspx
  The math input panel in Windows 7 uses the built in math recognizer to recognise mathematical expressions. It can be used with a tablet or tablet PC to insert equations into Word or MathType. If using OneNote to take handwritten notes using a tablet or tablet PC, you can convert equations in these notes to electronic format (English text can also be converted or searched if correctly recognised).

LaTeX packages of interest

- **The Extsizes package.**
  http://www.ctan.org/tex-archive/macros/latex/contrib/extsizes
  The Extsizes package provides classes extarticle, extreport, extbook, extletter, and extproc supporting normal font sizes of eight, nine, ten, eleven, twelve, fourteen, seventeen and twenty points. Other fontsizes in the document (e.g. the headings, footnotes and scriptsizes) are automatically scaled appropriately hence this allows a straight-forward approach to producing large or small print sizes from existing documents. It is noted that while LaTeX will re-flow English text there is not automatic re-flow of equations. Hence equations may need to be re-typeset.

- **The MH bundle.**
  http://ctan.org/pkg/mh
  The MH bundle includes the breqn package and packages required to support this. The breqn package can be used to write LaTeX in which displayed equations permit automatic line-breaking and hence can be used to assist with the creation of notes which can be increased in fontsize with greater ease.

- **The AMSLaTeX package.**
  http://ctan.org/pkg/amslatex
  The AMSLaTeX package provides a wide variety of small additions which can assist with the creation of large and clear print notes (particularly for instance where the English font and font style matters). This includes, for instance, typesetting (in the normal font) text fragments within equations and document level styling of theorem-like environments.

Key literature

Archambault, D. et al., 2007. Access to Scientific Content by Visually Impaired People. *UPGRADE The European Journal for the Informatics Professional*, VIII(2):29-42.

- Archambault et al provide a review of access to scientific work for people who are blind or partially sighted as of 2007. While now somewhat out of date this was a comprehensive review at the time and continues to provide a useful introduction. They conclude with remarks on the necessity for tools which allow for overview, collapse and expansion of elements of equations, synchronised representations to allow collaboration with sighted peers and colleagues and the need for technology to support the doing of mathematics.

Brinkmann, A., 2003. Graphical Knowledge Display – Mind Mapping and Concept Mapping as Efficient Tools in Mathematics Education. *Mathematics Education Review, The Journal of Association of Mathematics Education Teachers,* 16:39-48.

- Brinkman introduces mindmapping and concept mapping as natural methods to visualise interrelations between mathematical concepts. The strengths and limitations of these approaches in the specific domain of mathematics education are considered. This paper provides an introduction to using these approaches for mathematics and ideas as to how they may benefit all students for those wishing to explore this question.

Cardetti, F., et al., 2010. Insights regarding the Usefulness of Partial Notes in Mathematics Courses. *Journal of the Scholarship of Teaching and Learning,* 10(1):80-92

- Cardetti et al outline the results of an exploratory study into the use of partial notes in mathematics courses. A short review of the literature on note-taking is provided. In the study, partial notes were introduced into a course where no notes had previously been provided. Students in the second cohort perceived the partial notes to be beneficial to their study practices. An analysis of course examination results suggest that use of partial notes related strongly to high academic performance. The paper concludes with a discussion of the limitations of the study and implications for practice.

Cooper, M. et al., 2008. Access to mathematics in web resources for people with a visual impairment: Considerations and developments in an open and distance learning context. *Computers Helping People with Special Needs* 5105/2008. Lecture Notes in Computer Science, pp.926–933.

- A paper by Cooper et al which provides context for the paper by Cooper in this booklet. This paper is older but also fuller and with a focus on the needs of people with a visual impairment. Cooper's work focuses on the situation for web access rather than the wide picture and hence is likely to be of particular assistance when considering e-learning.

Easterbrooks, S., et al., 2006. Master Teachers' Responses to Twenty Literacy and Science/Mathematics Practices in Deaf Education, *American Annals of the Deaf,* 151(4):398-409

- This is one of the few studies I have found on good practice in Deaf education which specifically considers mathematics and in which the outcomes might be at least considered in higher education despite being based in the American school system. For instance, the use of visual organisers was the top ranked practice in science/maths. This is a teaching method both inclusive and still within the ability of a lecturer who may not be able to communicate directly with a student in their first language or without communication support.

Freda, C. et al., 2008. Dyslexia: Study of Compensatory Software Which Aids the Mathematical Learning Process of Dyslexic Students at Secondary School and University, in *Computers Helping People with Special Needs,* pp. 742-746, http://dx.doi.org/10.1007/978-3-540-70540-6_108

- Freda et al highlight the technological limits of word processing with speech, an assistive technology used by dyslexic students, which typically cannot assist with equations. This paper gives a very short introduction to planned research with the aim to produce a system which can be used with appropriate speech to support a dyslexic student in reading and writing mathematical text.

Hubbard, R., 1990. Teaching mathematics reading and study skills, *International Journal of Mathematical Education in Science and Technology* 21(2):265-269.

- Hubbard's argument for the specific inclusion of mathematical reading and study skills during the first year of a degree would appear to remain valid over twenty years on. Particular attention is paid to the importance of reading skills and it is noted that the skills required are different for mathematics texts. Hence, by reference to the specialist nature of such skills tuition it is concluded that this is likely to be the responsibility of the mathematics lecturer.

Jackson, A. 2002. Communications - The World of Blind Mathematicians. *Notices of the American Mathematical Society,* 49(10):1246–1251.

- Jackson introduces the reader to mathematicians who are blind, both in history but also to some researchers working today. As noted in the conclusion it is understandable that one might assume that technical notation may be a barrier but in contrast mathematics can be *more* accessible than other professions. This paper may be of interest to those considering mathematics as a future career and their advisors.

Jansons, K., 1988. Dyslexia and a mathematician's experience. In *Thought without language,* Weiskrantz, Lawrence, ed., Versailles: Oxford Science Publications.

- Jansons, a mathematician now retired, provided this personal account of dyslexia and the non-verbal reasoning he naturally uses in his work. Written in 1988 the account is still very relevant as I have found few of a similar nature. It may be of interest to dyslexic students who have a similar experience of language difficulties, mathematical thought and who are considering a career as a mathematician.

Karshmer, A. et al., 2007. *Mathematics and Accessibility: a Survey,* University of Texas at Dallas Technical Report, http://www.utdallas.edu/~gupta/mathaccsurvey.pdf

- Karshmer et al provide a comprehensive survey of mathematics and accessibility specifically for students with visual impairments. This survey is now somewhat out of date with respect to the particular technologies but can still provide a reader new to the area with context. The survey is particularly useful due to the clarity with which the varying approaches that might be taken by technologies both then and today are described. A focus on reading only is avoided as *working with mathematics* is considered. Finally the open problems as of 2007 are summarised. This technical document will still provide the reader with a foothold into the research area.

Maddox, S., 2007. Mathematical equations in Braille, *MSOR Connections* 7(2):45-48.

- Maddox describes the challenges faced by a UK Physics department in producing accessible notes when two students who are blind enrolled on their Physics degree. A very practical, if a little out of date, guide to a process, the successes and limitations, which nonetheless will raise questions for many departments. The partner paper by Whapples is mentioned below and together these may be seen as a contrasting approach to that described by Irving, Williams and Spybey in this booklet.

Mole, J. and Peacock, D., 2005. *Learning, teaching and assessment: A guide to good practice for staff teaching d/Deaf students in science and engineering.* University of Wolverhampton, http://www2.wlv.ac.uk/teachingdeafstudents/Science_engineering.pdf

- A very high level guide to good practice for staff teaching students who are d/Deaf. Some details are specific to science, engineering and also mathematics and these, explained in the context of more general guidelines may be of use. However, this guide does not look in any detail at the communication of mathematics to and by students who are d/Deaf and I have been unable to find any literature regarding this at higher education level.

Perkin, G. and Croft, T., 2007. The dyslexic student and mathematics in higher education, *Dyslexia,* 13(3):193-210.

- Perkin and Croft report on a 3 year research programme undertaken with dyslexic students in mathematical subjects. A broad introduction will benefit readers with little experience in the area. The case studies analysed present the difficulties dyslexic students can experience when learning mathematics. The cross-case analysis maps against content, delivery, procedures and processes, and assessment components to conclude that general barriers to success in mathematics do arise. Some recommendations are made.

Power, C. and Jürgensen, H., 2009. Accessible presentation of information for people with visual disabilities. *Universal Access in the Information Society,* 9:97-119

- Unlike the mathematics specific accessibility surveys mentioned elsewhere in the list this survey covers a wider context. However, unlike other general surveys the writers report extensively on both access to equations and to diagrammatic material within this wide context. Through a detailed and technical survey (up to date as of 2009) the reader is provided with a feel for the current trends and challenges. The paper concludes with a short list of areas in which further work is required by the community.

Stamerjohanns, H. et al., 2009. MathML-aware article conversion from LaTeX: A comparison study. *Towards a Digital Mathematics Library (DML 2009),* pp.109-120

- For some students presentation of resources in MathML may be beneficial. Hence departments may need to select a LaTeX to MathML converter. Stamerjohanns et al compare five LaTeX to XML transformers for usability, coverage and quality (particularly of the MathML). A list of systems *not* included in the comparison is given, with reasons, and readers will find this useful in determining the difference between tools presented on the W3C list. The five systems compared are Hermes, Tralics, LaTeXML, TeX4HT and TtM and these were tested using 1000 articles taken from arXiv.org. The analysis reveals something of the complex picture as of 2009.

Stanley, P., 2008. Assessing the Mathematics Related Communication Requirements of the Blind in Education and Career, in *Computers Helping People with Special Needs*, Lecture notes in Computer Science, 5105/2008, pp.888-891, http://dx.doi.org/10.1007/978-3-540-70540-6_133

- A very short and accessible high level article which captures the requirements from assistive technology of a person who is blind, working in a mathematical profession or studying a mathematical subject. Stanley concisely communicates possible barriers and then the attributes assistive tools require. This will be helpful to departments coming to this question for the first time and seeking a rapid overview to guide an exploration of assistive technologies which might be available.

Trott, C. and Perkin, G., 2005. Mathematical and Statistical Support for Dyslexic Undergraduates. *Helping Everyone Learn Mathematics Conference*, http://www.engsc.ac.uk/nef/events/helmconf_programme.asp

- Trott and Perkin outline some areas of difficulty in mathematics and statistics. These case studies are from one to one support sessions and some of the students are those reported on in Perkin and Croft's paper noted above. Trott and Perkin provide in this paper specific illustrations of some difficulties the three dyslexic students encountered alongside appropriate support methods and teaching materials. One of the students (studying a degree subject with only a minor statistical component) was also dyscalculic. This paper and others by Croft, Perkin and Trott provide a context for the paper Mathematics, Dyslexia and Accessibility contained within this booklet.

Trott, C., 2006. *Mathematics and Neurodiversity*, CETL-MSOR Conference, 2006, http://mathstore.gla.ac.uk/conference2006/Final%20Proceedings%202006/Conference%20Proceedings_WEB.pdf

- Trott provides a very high level summary of the mathematics support provided for students with a range of '*neurodiverse learning disabilities*' including dyspraxia, Asperger syndrome, ADHD, dyslexia and dyscalculia. A short section on mathematics support provided for a blind business student is also included. This paper will be useful to departments who need a rapid introduction to each of these areas with specific consideration as to the possible difficulties which may arise in mathematics.

Whapples, M., 2007. Obtaining Braille mathematical documents, *MSOR Connections*, 7(3):18-21

- Whapples describes the challenges faced throughout his studies in Physics in accessing notes and resources in Braille. A very practical explanation of the approaches used and challenges encountered. While technically a little out of date this paper should nonetheless raise questions for many departments. The partner paper by Maddox is mentioned above and together these may be seen as a contrasting approach to that described by Irving, Williams and Spybey in this booklet.

West, T. G. 1997. *In the mind's eye.* Prometheus Books New York.

- West describes his book as *"not intended to be a thorough academic analysis [..rather...] a kind of extended essay"*. In his essay West connects creativity, a facility for the visual rather than words and learning difficulties such as dyslexia. The first chapter overviews the area before he considers common experiences of dyslexia and some of the neurological basis. West goes on to profile eleven people, mostly well-known and considered remarkable in their field including three physicists, two mathematicians and two inventors. West argues that ten of the eleven show some form of pattern of learning difficulties and all are seen as highly creative with most being strong visual thinkers. He goes on to consider non-verbal thought, pattern, creativity and thus mathematics in this context.

Wigmore, A. et al., 2009. TalkMaths': A Speech User Interface for Dictating Mathematical Expressions into Electronic Documents," in *2nd ISCA Workshop of Speech and Language Technology in Education (SLaTE 2009), International Speech Communication Association (ISCA)*

- Wigmore et al describe the recent work at Kingston University to develop a speech-drive input system for mathematics which works with Dragon Naturally Speaking. They review the existing systems before considering the challenges of speaking mathematics unambiguously and that of understanding natural spoken mathematics. They describe their prototype system TalkMaths and their plans to improve on the system.

Courses with substantial mathematical content pose specific accessibility challenges beyond those usually considered in generic inclusive curricula good practice advice. This guide draws on knowledge and experience from academic staff, professional support staff, disability researchers and students. Contributions explore technical and pedagogic barriers and the way these may be formed by the modes in which mathematics is communicated. The contributions provide strong evidence of the need for collaboration between the MSOR community and the support professionals in dissolving barriers and moving together towards the goal of inclusive curricula.

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Emma Cliffe is Chair of the Accessing Maths, Stats and OR (AccessMSOR) working/special interest group of the MSOR Network. A mailing list provides a forum for members of UK HE institutions with expertise or interest in accessibility and issues surrounding supporting disabled students in MSOR subjects. Questions on aspects of MSOR access are welcomed and members are encouraged to share good practice, advances in technology and relevant news, workshop information or calls for conferences/participation. This is available via **www.jiscmail.ac.uk/accessmsorwg**