ltsn MathsTEAM

http://www.ltsn.ac.uk/mathsteam

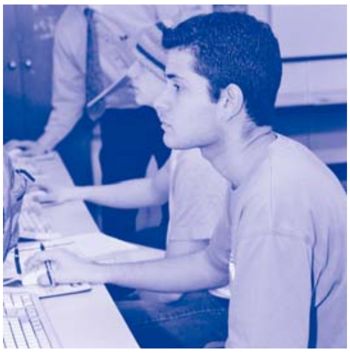
maths for engineering and science



maths for engineering and science

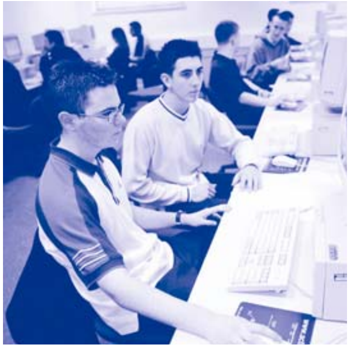
published by the LTSN MathsTEAM Project

Contents



Foreword

The appropriate education of engineers and scientists is an important element in the economic well-being of the United Kingdom, in common with other industrialised countries. Within that education the mathematics component has a central role.



For many years concern has been expressed about the decline in the mathematical skills possessed by entrants to engineering and science degree programmes. Students of today perform less well on diagnostic entry tests than those with apparently similar qualifications from the cohort ten years earlier. On its own this decline in key mathematics skills even amongst students who obtained reasonable A level grades would be a significant concern. However, the problem has been deepened by other trends in higher education during the 1980s and 1990s which resulted in a widening of the educational background of entrants to these programmes; the implication of this needs to be appropriately addressed. The growth in the numbers entering higher education has resulted in some students who are less well-qualified starting courses to which, previously, they would not have been admitted.

Whilst some subjects, such as psychology and business studies, have been able to maintain or even increase their required entry qualifications, engineering and science have become increasingly less popular and, consequently, have struggled to find sufficient numbers of recruits with the desired level of entry qualification. Any initiatives which increase motivation among students and give them alternative environments in which to improve their mathematics skills are to be welcomed. There are currently a number of innovative teaching methods being pursued, for example the use of different technologies, streaming or interactive lecture formats.

Other initiatives such as the introduction of problem-based learning are at too early a stage to be included, but this booklet provides an opportunity to share some initiatives with colleagues who are facing the same challenges with their own students. The outline case studies it contains merit scrutiny; every reader should find a number of them of interest. One or more of them may be of particular appeal and the authors concerned may be contacted for further details.

The LTSN MathsTEAM is to be congratulated on its foresight in commissioning these case studies and putting them in booklet form. Members of the team will be happy to assist you with any further information which you may require. Maths for Engineering and Science is a valuable contribution to the work of those of us who teach mathematics to engineering and science students and I recommend it to you for serious consideration.

Leslie Mustoe
Science and Engineering Foundation Studies, Loughborough University.

LTSN MathsTEAM Project

Funded by the Learning and Teaching Support Network (LTSN), the LTSN MathsTEAM Project (**http://www.ltsn.ac.uk/mathsteam**) has carried out an in-depth survey, examining the following three topics:

- **Maths support programmes and resources,**
- **Current practices for teaching mathematics to engineering and science students,**
- **Diagnostic testing.**

The information has been published in three booklets:

- **Maths Support for Students,**
- **Maths for Engineering and Science,**
- **Diagnostic Testing for Mathematics.**

They each provide a comprehensive collection of case studies, intended to assist you with the challenge of enhancing the basic mathematical skills of engineering or science students.

The contributing authors discuss the execution of current teaching practices based on each of the three topics mentioned above. They talk about the barriers and the enablers in setting up these learning initiatives. For those of you considering the implementation of any of the programmes, each case study provides an opportunity to review the learning processes and tools involved. Each booklet contributes to the transfer of knowledge within higher education communities; each case study offers practical suggestions for you to gain a better understanding of the present situation and related topics that merit further exploration and research.

The UK Mathematics Learning Support Centre

During recent years, throughout the higher education community there has developed a growing need to share knowledge and materials, to develop good practice and stop re-inventing the wheel. Funding has recently been made available for the development of the first UK Mathematics Learning Support Centre – mathcentre.

The Centre will use a mix of modern and traditional techniques to allow both students and university professionals free access to samples of high quality learning materials aimed at alleviating the school/university interface problem. It will also use the resource base created by such projects as the LTSN MathsTEAM.

Further information about the mathcentre can be found at **www.mathcentre.ac.uk**.

Maths for Engineering and Science

In September 1999, the Institute of Mathematics and its Applications (IMA) made the recommendation that, in both the Incorporated Engineers (IEng) and the Chartered Engineers (CEng) programmes, mathematics topics should be taught within an engineering context (IMA, 1999).

Implementing this recommendation requires knowledge of the teaching methods that are currently being used in the engineering community. The LTSN MathsTEAM has brought together this information as well as other relevant information from the science community.

For many academics, technology forms the basis of the contextual approach. For some the approach is ‘fun’, as one lecturer describes the teaching of a mathematical modelling course taught to first year chemical engineers. Others have introduced streaming so that the contextual approach can be tailored to students with different mathematical skills.

Such innovations in learning and teaching methodologies are becoming more important as academics face the challenge of teaching science and engineering students with a wide diversity of prior knowledge.

This booklet contains structured case studies from contributing authors describing the execution of the learning activities, the support needed, the implementation difficulties, evidence of success, and suggestions of how other academics could reproduce the activity.

From foundation year through to final year, every one of the teaching methods focused on the needs of the students. Each illustrated that in developing mathematical thinking science and engineering students needed to be set meaningful tasks, but tasks that were so structured that they were accessible to both the weaker students and the more able. The aim was to create good practice, which would engender mathematical thinking.

The booklet offers you a chance to explore the growing diversity of context based initiatives through these examples of good practice found within Higher Education institutions throughout the UK.

Reference: “Engineering Mathematics Matters”, published by the Institute of Mathematics and its Applications, September 1999.

LTSN Maths TEAM Project I Maths for Engineering and Science

Setting the Scene

The Mathematical Attitudes, Beliefs and Ability of Students

Paul Ernest ▪ School of Education and Lifelong Learning ▪ University of Exeter

There is growing evidence of the importance of students' attitudes and beliefs about mathematics for their achievement in and successful applications of the subject [1]. Research studies have shown that students in higher education who are not maths majors often have negative images, beliefs and attitudes towards mathematics [2]. There is great variation across all students, especially with engineering students, who can be mathematically very strong through to some who are quite weak. It is often but not invariably the case that mathematical achievement is correlated with positive attitudes to the subject. Typically, it is confidence in one's own mathematical ability that is correlated with achievement rather than liking or pleasure in the subject.

Where such correlations do occur it is observed that the achievement-attitude link forms self reinforcing cycles [3].

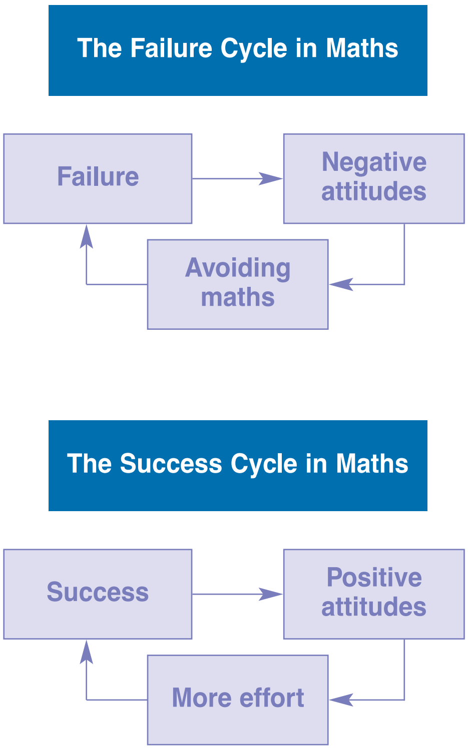


Figure 1: Failure and Success Cycles in Maths

Low achievement or repeated failure in maths often leads to negative attitudes and lowered confidence, resulting in reduced effort or even maths avoidance, leading to further failure. This is a vicious cycle. Engineering students are likely to have strong overarching goals concerning success in studies and may refuse to allow any developing negative attitudes to maths to impede their learning efforts. However in such circumstances their beliefs about maths will tend to be that it is simply a toolkit, a 'necessary evil' required for their overall success. Nevertheless a minority of students caught in such a cycle may be discouraged enough to give up their studies.

Positive achievement and success in maths often lead to enhanced attitudes and raised confidence, resulting in increased effort and persistence, and further success. Many engineering students will have a history of success and achievement in mathematics behind them as this is normally an entry requirement for engineering courses. But there is no simple pattern to the beliefs about maths of engineering students, for in addition to what they bring with them on entry, their learning experiences on their university course will do much to shape their beliefs about mathematics.

Mathematics for engineering students at university has in the past typically been made up of service courses provided by the mathematics faculty. A 1988 survey of 60 engineering departments in the US found that most were happy with the courses provided by maths departments. However, the mathematicians did not seem to have a favourable attitude to the engineers, who in turn opted only to take the minimum number of courses necessary [4]. Many mathematics staff dislike service teaching. Courses often have overloaded syllabuses, are far from mathematicians' research interests and are made up of large lectures with many apparently disinterested students [5]. A key question is "should the way to teach mathematics to engineers be different than for pure maths students?" [6].

Two sets of contrasts can been drawn: 1) between mathematics taught for its own sake or as a service subject, and 2) between maths taught as a separate subject and integrated into other studies [7]. Traditionally mathematics for engineers is taught separately as a service subject. While not all students react in the same way to this experience, a number of studies report that engineers subsequently view maths as a toolkit, the application of which needs to be learned almost 'by heart', and they also have difficulty in using maths in relatively low-level problem solving and non-routine engineering applications [8].

Thus a common belief about mathematics that emerges from such experiences is that it is an isolated set of abstract ideas, with seemingly little relevance to applied problems, and comprises a set of tools whose applications need to be mastered individually for every context or type of problem. In other words, this knowledge seems to lack ease of transfer to new problems and general applicability. This is not healthy for future engineers. Such students also share the perception that it is a 'cut and dried' set of rules and procedures which provide a single or best way to formulate and solve mathematical problems.

However, teaching mathematics separately as a service subject does not have this impact on all engineering students. A minority with a strong grasp of mathematical concepts and principles clearly benefited from this approach and subsequently viewed maths as a powerful and well integrated discipline with broad and general applications. Such students see mathematics as separate from applications, but recognise the multiplicity of possible solutions to engineering problems [9].

There are a number of experimental practices in Denmark, UK and USA, for example, in which mathematics is taught integrated into engineering and other applied studies. It is argued by proponents of such an approach that the development of a feeling for mathematics and mathematical common sense in such applications is much more important than mathematical rigour. These capabilities aid the integration of mathematical knowledge and skills into engineering and lay to rest the commonly expressed doubts about the value of certain mathematical topics studied in isolation [5].

Integrated approaches are typically based on modelling exercises which can be described as simulations and case studies. Although such courses initially cause bewilderment and confusion among traditionally taught students, because of their open-ended tasks and lack of specific directions, by the end of the course most students have superior confidence and some creativity in applying mathematical knowledge [10]. When asked the differences between such approaches lower level students often focussed on two features: the validity and acceptance of multiple solutions methods and the lack of need to remember by heart a single 'correct way'. These students also reported growing confidence in their mathematical skills in a supportive and non-threatening environment in which seeking to find the unique correct way was not the main goal. Mathematics was "a challenge and not a problem" [11].

Thus there is some evidence that an integrated or modelling approach to mathematics for engineering students leads to improved attitudes and more helpful beliefs about mathematics. By improving application skills and fostering teamwork and communication skills (these are often integral to integrated courses) such approaches may address the often heard concerns of employers that graduates have a relative lack of presentational and communication skills, a lack of pragmatism, business and engineering skills and inexperience in some contexts [12].

Such findings and recommendations must be viewed with caution. Clearly engineering students' attitudes and beliefs about mathematics are important. Not only can they sustain them through their studies and have an impact on their willingness to engage in further studies post-graduation to continually upskill themselves during their professional careers. But also they can impact on their ability to successfully apply their knowledge to engineering tasks and problems.

However there is no guaranteed method of course delivery that can guarantee to deliver these outcomes. A well designed and taught service course in mathematics may be more effective than an integrated course of mathematics for engineers if the latter does not reinforce, draw out and make explicit the key concepts and methods in practical engineering applications. Furthermore it is unlikely that all mathematical needs can be addressed this way, for there will always be a need for the mastery of a repertoire of skills and methods. Nevertheless, such integrated modelling based courses appear to be a fruitful avenue for further development and experimentation.

References

[1] *The Philosophy of Mathematics Education*, Falmer Press, London, Ernest, P., (1991).

[2] *Adults Mathematical Thinking and Emotions*, Falmer Press, London, Evans, J., (2000).

[3] *Mathematics and Special Educational Needs*, University of Exeter, Ernest, P., (2000).

[4] *Teaching Mathematics as a Service Subject*, Siegel, M.J. in [9] pp75-89.

[5] *Teaching First Year Students* Simons, F. in [9] pp35-44.

[6] *Mathematical Education for Engineering Students* Murakami, H., in [9] pp63-69.

[7] See both the report in *Proceedings of Adults Learning Mathematics Conference*, Exeter, Wedege, T., (1995) and *Some Reflections about the Teaching of Mathematics in Engineering Schools*, Roubine, E. in [9] pp70-74.

[8] *International Journal for Mathematical Education in Science and Technology* 14(2), p217, Berenson, L. and Robinson, B., (1983).

[9] *Mathematics as a Service Subject*, Howson, A. G., Kahane, J-P., Lauginie, P., Turckheim, E. (eds) Cambridge University Press, (1988).

[10] One such study at Bristol University Department of Engineering Mathematics is reported by Clements, R.R., *Teaching Mathematics to Engineering Students utilising Innovative Teaching Methods* in [9] pp45-57.

[11] *Proceedings of Adults Learning Mathematics Conference*, Birmingham: reporting experimental work at Rutgers University in the USA, Safford, K., (1994).

[12] *Report of the MathsSkills Discipline Network*, University of Hull, Kopp, E., (1998).

The Changing Relationship: Civil/Structural Engineers and Maths

David Blockley and Norman Woodman ▪ *Department of Civil Engineering* ▪ *Bristol University*

Published in *The Structural Engineer – 2nd April 2002*

Synopsis

Mathematics is vital for civil engineers but its role is changing. Arup chairman Duncan Michael [1] has argued for less emphasis on the teaching of mathematics. Here we report on a necessary change of emphasis but also argue the importance of a good mathematical education for all engineers.

Introduction

We presume that when Duncan Michael justifies his statement that we should teach far less mathematics to our young engineers by saying that 'anyone over 50 today is unlikely to be able to break out sufficiently from their acquired beliefs and presumptions' he includes himself! In our experience, this ability to adapt and change is less to do with actual age than with attitude. We have met young people who are old in this respect and old people that are young! Certainly, those in University research and education must keep young! As the first author has recently written a text Doing it Differently [2] we feel in a good position to discuss Duncan Michael's case even though we are both over 50!

His assertions are nothing new. When three mathematicians were asked in 1742-3 by Pope Benedict XIV to examine the cracked dome of St Peter's their report was severely criticised at the time. 'If it was possible to design and build St. Peter's dome without mathematics and especially without the new fangled mechanics of our time, it will also be possible to restore it without the aid of mathematicians …… Michaelangelo knew no mathematics and yet was able to build the dome' [3]. There have been innumerable statements by people rejecting the need for mathematics but time and time again history has shown them to be wrong. What is esoteric and complex in one era may become commonplace in the next.

However, there is a major difference in modern times and that is the power of modern computers and this has a serious effect in the way we think and use mathematics. No longer do we have to 'plough through' long pages of deductive proof – the computer will do it for us. No longer do we have grind through long calculations – the computer will do it for us. The challenge has changed from the ability to do this to the ability to interpret the meaning of mathematics to engineering and herein lies the challenge and change of emphasis.

Did you Learn Tables at School?

When electronic calculators were first available the teaching of tables was abandoned by many schools. At that time the first author came across a 15 year old who knew what multiplication was but not what the answer to 12x5 was without a calculator. However she knew to write 5 down 12 times and add them up! Fortunately many schools now teach tables as well as the use of calculators.

Those of us over 50 were generally taught mathematics purely on the basis of it being a tool for calculation.

Mathematics is a Language of Scientific Communication

If you were dropped off somewhere remote in France and told to find your way to somewhere else rather remote you will certainly find it easier if you speak French. Of course you may probably succeed without being able to speak French at all but it may take you longer or you may encounter other difficulties. The same is true of mathematics because it is the language of scientific communication. Without a facility in mathematics you cut engineers off from scientific change and development. Engineers so often confuse the science with the language and what is being rejected is not mathematics per se but inappropriate theory. There is a place for all levels of theory.

The understanding of a physical phenomena such as structural behaviour is very important – but that is no reason to reject mathematics. Many of us learned how stiffness attracts moment through many many moment distribution calculations. This however is not sufficient to justify teaching moment distribution when it is so much easier to use a PC. However how do young people learn about stiffness and moment? – that is the challenge to modern teachers of structural analysis. All of those matters learned by grinding through lots and lots of examples have now to be learned more efficiently – but how? One thing is clear: if an engineer is 'blinded' by an inability to understand the language of a book or technical article then important engineering phenomena may well be misunderstood or missed completely.

Mathematics is About Rigour

Many of us over 50 enjoyed Euclidean geometry and the beauty of theorem proving. This is no longer in the syllabus. However all mathematics is the ultimate form of logical rigour. This is certainly a quality required of engineers. The over concentration on getting the 'right answer' in a mathematical question at school has been to smother creative thinking in many people but one must be careful not to throw the baby out with the bathwater. In these modern times when people are increasingly relying on 'bullet point' presentations the ability to work through a set of ideas using a strongly logical mind is of very great importance. We also find that the preparation of engineers in Europe is to a level of mathematics that our students find it hard to compete with – we must be able to compete with the best in the world.

Mathematics Is a Dense Language

We learn mathematics sequentially and we gradually build layers of understanding. You cannot dip in and out of mathematics. This makes it inhibiting to many because unless you have understood the lower layers you cannot hope to understand the higher ones. Thus many become frightened of mathematics. The language is dense and hard – but is essentially why it is educationally important to train the minds of our young people. We need engineers who are at ease with it and who can take advantage of new ideas and use them appropriately even if they are expressed using advanced mathematics. Technician engineers will need less mathematics than chartered engineers but both need to be comfortable with an appropriate level of skills for the responsibilities they take on.

Computation has Over Taken the Calculation and Deduction

Thus the new emphasis is on modelling. If we have a problem – a structure to design – then modelling is about 1) building an appropriate theoretical model, 2) deducing some results from it and 3) interpreting those results into decisions regarding your structure. The model may be a physical model, say of cardboard, to examine how the structure may be assembled. The model may be a theoretical model based on physics but expressed using mathematics. Traditionally almost all of the focus of engineering education has been on the middle step that involves mathematical manipulations. Now it is on steps one and three. In that modelling our scientific understanding of physical (and human systems) is crucial. Intuition can be wrong – science is about making our models objective (i.e. describing things in a way that can be shared by others [4]), testing those models and updating them. We get dependable information when we can measure it (necessarily involving mathematics) and when we can subject it to intense scrutiny from all angles.

Examples of Pitfalls

We will now quote just two examples of where inadequate understanding of science and mathematics can lead us astray.

Finite element approximations

Our colleagues have experienced many examples of practising engineers making wrong assumptions in finite element modelling. It is very easy to get the right answer to the wrong problem. All packages have limitations and we have experienced examples where engineers have set, as boundary conditions, degrees of freedom that were not present. Many engineers seem not to realise that using grillages to model a slab will not adequately model torsion. A Chartered engineer must have a strong understanding of the analysis packages used and this requires an adequate level of mathematical knowledge.

Reliability Theory

Few practising engineers have a good grounding in probability theory. In the minds of most engineers probability theory is synonymous with statistics. They know their business is one where data is sparse. Hence statistics and probability theory are dismissed by the vast majority of engineers as being of little interest. Over many years mathematically inclined engineers have developed reliability theory based on the use of probability theory. Where data is sparse they use the so-called Bayesian approach that uses subjective judgement in a very special but rigorous way. Few engineers are able to criticise the approach adequately because of their inadequate understanding of probability theory. Thus the problem of incompleteness in reliability calculations written about extensively by the first author are not appreciated. This is very dangerous since risk numbers are used by some engineers that are totally spurious. Here is an example where modern mathematics should be taught to engineering undergraduates so that they have the theoretical understanding to address one of the most basic issues in modern society – the way we handle risk.

We Believe we Need a Systems Approach

In his recent presidential address to the Institution of Structural Engineers [4] the first author has argued for a systems approach to engineering. This is the real paradigm shift that we think Duncan Michael should focus on. In our research using mathematics we have produced several new approaches including two new theories from using this thinking – structural vulnerability theory and the Interacting Objects Process Model [5,6]. Rather than deprecating the use of mathematics which will reduce our capacity to develop new ideas we should be looking for new ways of using it where it is appropriate and teaching our young engineers to understand and to use the language of mathematics in their qualitative and quantitative work.

References

[1] *Civil Engineers and Maths*, Civil Engineering, 150(1), Feb, 8,9., Michael, D., (2002).

[2] *Doing it Differently*, Thomas Telford, London, Blockley, D. I., Godfrey, P. S., (2001).

[3] A History of Civil Engineering, translated from *Die Geschickte der Bauingenieurkunst*, 1949 by Rockwell, E., Leonard Hill Ltd, London, Straub, H., (1952).

[4] 'Thinking out of the Box', *The Structural Engineer*, 79(20), 16th Oct, p22-29, Blockley, D. I., (2001) (Presidential Address to IStructE, 4th Oct 2001).

[5] 'Structural Analysis using the IOPM', *Advances in Eng'g Software*, 28, p293-301, Argarwal, J. A., Blockley, D. I., Woodman, N. J., (1997).

[6] 'Vulnerability Analysis of 3D Trusses', *J. Struct Safety*, 23(3), p203-220, Argarwal, J. A., Blockley, D. I., Woodman, N. J., (2001).

Approaches to Teaching for Engineering and Science

Teaching Students with Diverse Backgrounds

Teaching Mathematics to First Year Engineering Students with a Wide Range of Mathematical Ability

Tim Bullough ▪ Department of Engineering ▪ University of Liverpool

Abstract

The approach to teaching Maths to Year 1 students in the Department of Engineering underwent a major reorganisation prior to the start of the 2002/3 session. The aim was to provide an optimum framework within which students studying four different engineering disciplines could be taught Maths within the resource constraints imposed by student numbers, and to cope with the extremely wide range of their Mathematical abilities on entry to these degree programmes. After much discussion, students are now taught their Year 1 Maths topics in two different cohorts, streamed according to initial Maths ability, and using different approaches in terms of the depth of understanding expected. This also involves the use of different assessments. This approach has been much more popular and created far fewer difficulties than the previous system which divided the students into two groups according to degree programme.

Level of Material: First Year

The Problem

Mathematics has always been taught to Year 1 engineering students in the Department of Engineering at the University of Liverpool by lecturers from the Department of Mathematics. The Department of Engineering currently supports four accredited Engineering degree programme groupings: Aerospace Engineering, Integrated Engineering, Materials Engineering and Mechanical Engineering. Numbers have always fluctuated, but until recently there has been roughly the same number of students studying Mechanical Engineering as taking all the other three programmes in total. For at least the past 10 years, all Mechanical Engineering students took a very traditional rigorous Year 1 mathematical module, which assumed a good A-level Maths grounding (grade C or above was assumed). Aerospace Engineering, Integrated Engineering and Materials Engineering students all took a different Maths module. Students were therefore divided for their Maths teaching in Year 1 by degree programme.

In recent years, Maths entry qualifications for these four groups of programmes has been spread quite evenly between Maths A-level grades A-D and Foundation Year/other qualifications (including overseas). However, whereas many Aerospace Engineers had good Maths entry qualifications, a significant minority of students were entering with Maths qualifications towards the lower end. There was anecdotal evidence, supported by Maths diagnostic testing at entry (introduced for the 2001/2 session), that these students were not comfortable with basic techniques of algebraic manipulation etc. The Year 1 Mathematics module delivered to Aerospace, Integrated and Materials Engineers had initially adopted the same rigorous approach as the one delivered to Mechanical Engineers, but it was having to continuously adapt to include more remedial aspects of Maths topics in order to give the weaker students any chance of success. In essence, this meant more teaching of basic mathematical techniques, including algebraic manipulation and the application of mathematical formulae, and less rigour in terms of understanding the underlying mathematical processes (which was identified as important for Engineering students later in their courses). While this approach was successful in terms of helping the weaker students, some of the better qualified students were increasing becoming bored as they were not being stretched or stimulated, and were potentially being disadvantaged.

For the start of the 2002/3 session the whole approach to teaching Maths to Year 1 students in the Department of Engineering underwent a major review, both in terms of content and delivery. The only resource constraint was that two Mathematics teaching staff remain available for the teaching of the Year 1 cohort of typically 120-160 Engineering students in total. This review also coincided with the Department adopting a ~95% common Year 1 structure for all their BEng and MEng programmes.

The Solution

There were three main approaches, discussed in detail by Engineering programme directors and Maths teaching staff, in terms of how to better organise the teaching of Year 1 Maths:

1. Retain two Maths modules and split the students into two groups based on degree programme (as previously).

Main advantage: the Maths content and engineering applications used in teaching the Maths could be tailored for the specific engineering discipline, although with the introduction of a common Year 1 structure this was not felt to be very important.

Main disadvantage: it would retain the very wide spread of Maths ability within both groups.

2. Retain two Maths modules, both covering the same Maths topics, and split the students into two groups based on Maths ability (including their prior Maths qualification).

Main advantage: it would allow better students to be stretched (reducing the likelihood of them getting bored) and permit remedial teaching to the other group as necessary.

Main disadvantage: a process for dividing the class into the two groups would need to be agreed with staff and students, which optimises the learning process for each group, without allowing any suggestion that one group is taking an "easier" or a "harder" module.

3. Teach all students in a single large group, but run parallel remedial support sessions as necessary.

Main advantage: Uniformity of teaching style, content and assessment.

Main disadvantage: the Maths teaching staff did not think it would be possible to provide suitable content, delivery and assessment for the very wide spread of Maths ability in the group. Maths staff also felt that students who require optional remedial support are often the ones that do not take advantage of it. Remedial support can also be provided equally well within the other approaches.

After much discussion, approach 2 was agreed i.e. Year 1 students were "streamed" into two different groups, with both groups taught the same topics: Vector algebra, Differential Calculus, Functions of two variables and partial derivatives, Complex numbers, Integration and Differential Equations (Note: matrices taught in separate module). The division into groups was on the following basis:

- All students with Maths A-level grades A-C (about 70%) take a "standard" Year 1 Maths module. This uses what could be called a "rigorous" approach to teaching, including coverage of the underlying mathematical methodologies (and no access to formulae sheets for assessments).
- All remaining students (Maths A-level grade D, non-A-level qualification, ex-Foundation Year, and overseas students) initially assigned to a second Maths module. Primarily students are taught to use maths formulae to solve problems (with a formulae sheet provided for use in all lectures, class-tests, homework and examinations). While both modules utilise the same number of lecture hours, a weekly tutorial session is made compulsory for all students on this module.

Student performance in their start-of-year Maths diagnostic test, their first class-test around week 5, and (where applicable) their Foundation Year Maths module marks were all used to permit a small number of students to be moved "up" to the "more rigorous" Maths group in the first few weeks. Thereafter no movement between groups is permitted.

The most controversial aspect of the arrangements was assessment. Because the approach to teaching was so different, it was finally agreed that the two modules must have DIFFERENT assessments (i.e. different types of questions in their class-tests and exam papers) – although the re-sit exam paper each year will be common for both streams, including the provision of a formulae sheet. It was also agreed that, in order to overcome any perception by students that they may be disadvantaged by taking a "harder" module, the teaching staff will co-ordinate their assessment processes to ensure that students taking the "rigorous" maths module do not fail the module if they would have passed the techniques-based maths module taken by students with weaker Maths qualifications. It was also made very clear to students that although both modules provide an adequate solid training in the essential Mathematical techniques that will be required of Engineering students in their second year, the "rigorous" maths module was more appropriate for Engineering studies if it was felt that they could cope with the rigour.

The Barriers

There were a lot of entrenched attitudes amongst teaching staff on all sides, ranging from extreme views that absolutely no compromise should be made to students with weaker maths backgrounds, through to serious concerns that with there being two differently assessed modules, no student should feel they are disadvantaged.

The Enablers

It was made very clear to students that although both modules provide a solid training in the essential Mathematical techniques that will be required of Engineering students in their second year, the "rigorous" maths module was the preferred one for Engineering studies if students could cope with it.

Evidence of Success

As of now, we only have initial feedback from staff-student forums and tutors on student reactions to this teaching approach, and these have been universally positive. As the main assessment and formal student feedback is only obtained at the end of the year, in the short term attendance at lectures and performance in class-tests will be used as a guide. From our viewpoint, we would like to know whether the new approach provides students with a more positive attitude and increased confidence towards the use of Maths in their engineering modules. This is difficult to assess!

How Can Other Academics Reproduce This?

Discussions have already been initiated to ascertain whether we may be able to widen the dual approach to Year 1 Maths teaching to include other engineering degree programmes which have a similar wide spread in Year 1 Maths ability, but insufficient student numbers to justify the allocation of more than one Maths lecturer.

A willingness of staff from both Maths and Engineering to devote a considerable amount of time (and make considerable compromise) was required in order to come up with a mutually agreeable outcome.

Quality Assurance

In situations where University policy requires that Maths is taught to engineering students by staff from a Maths department, it is important that procedures exist which encourage regular discussion between Maths and Engineering staff. This was not always the case in the past.

We now have a single identifiable member of staff in the Department of Maths with responsibility for Maths teaching to our Department and he attends Engineering staff-student meetings and programme review meetings.

Teaching Students with Diverse Backgrounds

Streaming Undergraduate Physicists for Mathematics Teaching in Year One

Mike Savage ▪ Department of Physics and Astronomy ▪ University of Leeds
Tom Roper ▪ School of Education ▪ University of Leeds

Abstract

Many departments of mathematics, physics and engineering now use some form of diagnostic test to assess the basic mathematical skills of new undergraduates [1]. Results reveal that a typical cohort consists of students with a diverse range of mathematical backgrounds and capabilities. Tests also help to identify those students who lack both confidence and competence and are deemed to be at risk of failing or dropping out in Year One.

It is now commonplace for those teaching first year mathematics to be faced by an inhomogeneous student cohort and all are in accord that it has become almost impossible to teach them effectively together. It is against this background that streaming of first year undergraduate physicists into two more homogeneous groups has been introduced at the University of Leeds. The aim is to provide more effective teaching and mathematics support that will get students up to speed and mathematically prepared for their second year.

Level of Material: First Year

The Execution

The case study begins in Intro Week with a diagnostic test. Subsequently students are streamed for all first year mathematics modules.

Intro Week: Students take a written diagnostic test during Intro Week to assess their basic mathematical capabilities in arithmetic, basic and further algebra, trigonometry and calculus. They are then allotted to either Group A or Group B on the basis of their test performance. There is negotiation with students who are close to the borderline to decide which group might be most suitable for them.

Modules: In Semester 1, students take both a 5 credit and a 10 credit module

Maths Consolidation, (5 credits), 1hr/week

Maths for Physicists 1, (10 credits), 2hrs/week.

In Semester 2, there is one 10 credit module

Maths for Physicists 2, (10 credits), 2hrs/week.

Remarks:

- In Semester 1, there are 3hrs/week for mathematics teaching. Group B uses all 33 hours whereas Group A uses 30 hours at most.
- Printed notes are used, enabling students to engage in discussion and in doing examples during lectures.
- The first four weeks are given to reviewing and consolidating basic A-level Maths skills.
- An additional “Booster Maths class”, 1hr/week, is provided for specific students needing extra teaching/practice with basic skills.
- Students attend a weekly examples class, with attendance compulsory and at most 30 students in each class.
- Module assessment is via marked assignments and a written examination that contribute 15% and 85% respectively.

A major aim of the first year mathematics programme is to engage students in doing mathematics by confronting and solving problems. It is made clear to students at the outset that attendance at examples classes is compulsory and assignments are to be handed in regularly and on time. Of central importance, therefore, is a procedure for continuously monitoring students’ attendance, attitude and performance.

Students who miss consecutive classes and/or fail to submit work are asked to explain their record to the Year Convenor and in some cases with the Head of Department.

Pre-requisite Knowledge

The only requirement is for students to have at least a grade D in A-level mathematics. In our experience the A-level grade itself is an unreliable indicator of a student’s mathematical knowledge and ability. The function of the diagnostic test is to provide a profile of what each student can and cannot do.

How Are Students With Different Mathematical Backgrounds Supported?

The function of ‘streaming’ is to provide an effective support mechanism for students from different mathematical backgrounds with different levels of preparation. The Booster Maths class gives additional mathematics support to those who most need it and are considered to be at risk.

What Support Was Needed?

It is essential to have the support of the Head of Department and other members of staff, all of whom recognise that:

- Mathematics is the modelling language for physics and engineering.
- Students who are mathematically ill-prepared on entry need special attention in order to bring them up to speed in Year One.
- Good mathematics teaching in Year One is essential since many students have been 'put off' mathematics during their school experience. Both lecturers and the Booster Maths tutor have considerable experience teaching mathematics to (a) sixth formers (TR) (b) Open University students (MDS, SW) and (c) first year undergraduates (MDS, TR, SW).

The Barriers

A major obstacle is the lack of motivation in students who fail to see the purpose of mathematics. In fact many academic physicists have reported [2] that their new undergraduates regard maths and physics as two different disciplines. In particular they fail to see mathematics as the modelling language for both physics and engineering and this is, in part, due to a trend over the past decade to reduce the use of mathematics within A-level physics. Hence teachers of mathematics to first year undergraduates in physical or engineering science have a doubly difficult task. On the one hand they must ensure that the basics are understood and well practised whilst on the other, they have to motivate each mathematical topic by illustrating how it connects with ideas and topics within their physics and engineering courses.

A second obstacle is the reason frequently given for not introducing streaming, namely the non-economic, additional cost of employing a second teacher. Superficially this may appear a compelling reason for maintaining the traditional teaching format. However, recent experience at Leeds has shown that this initial, extra investment can be repaid several times by substantially reducing the drop out and failure rates in Year One – i.e. streaming can be a cost effective policy.

The Enablers

Specific strategies for promoting student participation are

- When referring to Group B students the Maths Team and the Department are careful not to use emotive terms that could label them as second-rate. These students are simply recognised as being ill-prepared at the outset and so receive teaching appropriate to their needs. Otherwise the two groups are treated exactly the same; they do the same course, they are assessed in the same way and high achievements are expected from both by the end of year one.
- The provision of a Booster Maths class for students at risk.
- The provision of examples classes, each with under 30 students, staffed by lecturers and postgraduates who offer help and encouragement.
- The provision of a 5 credit, super-numeracy Maths Consolidation module in the first semester. This is found to be very effective in engaging students early on to consolidate and develop their algebraic, trigonometric and calculus skills. The module is tested in week 8 (pass mark 70%) with a re-sit in the normal examination period in January. As a result, students are able to see before the Christmas vacation how much progress they have made with basic skills since the diagnostic test!
- The introduction of streaming in all first year maths modules. Though some progress is made in Semester One, it is in the second semester that the real effect of streaming becomes apparent – when Group B students have gained substantial confidence and can produce good and in some cases excellent examination performances.

Evidence of Success

The success of the case study is demonstrated via

- Student feedback (module questionnaires).
- Attendance at examples classes and submission of assignments.
- Examination results.
- Reduced drop out and failure rates.

How Can Other Academics Reproduce This?

Streaming for first year mathematics teaching could be applied to students in mathematics, physical science, engineering or indeed any maths based degree scheme.

Quality Assurance

All modules and courses in Physics and Astronomy undergo annual review where feedback on teaching and assessment is examined. The QAA review panel (2000) commented favourably on the department's provision of mathematics support to first year students.

Other Recommendations

A necessary requirement for the case study to succeed is the setting up of an effective Maths Team that is fully attuned to the Department's aim and objectives for first year mathematics teaching and support.

References

[1] *Measuring the Mathematics Problem*, Engineering Council, London, Hawkes, T., and Savage, M. D., (2000).

[2] Professor Stuart Palmer, Guardian University Guide, 26 Oct, 1999.

Teaching Students with Diverse Backgrounds

A Learning Framework for Basic Mathematics and Statistics in Science

Graham Currell and Tony Dowman ▪ Faculty of Applied Science ▪ University of the West of England

Abstract

The teaching of mathematics and statistics forms part of a first year module, Scientific Inquiry, which is taken by students on undergraduate science courses. The range of mathematical backgrounds amongst the students is accommodated through a Virtual Learning Environment (VLE), supporting student-centred learning. The Case Study describes the philosophy of the module and how this is reflected in its structure, delivery, available materials and use of self-assessment.

Level of Material: First Year

The Execution

The core philosophy of the module design is that the learning provision should include:

- A clear demonstration of what the student is expected to achieve.
- Opportunities, within a credit-bearing context, for regular self-assessment of capabilities.
- A study programme which is appropriate for a range of entry skill levels.
- Study support materials available on-line through a VLE and the internet.
- Lectures to introduce new material and provide general guidance.
- Computer supported tutorials for demonstrations and individual support.

Students sit a paper-based, but computer-marked, diagnostic test (40 questions) during the Induction Week. The results, together with answers and interpretation, are returned to the students within the first teaching week. The aim is to introduce the idea of structure within the learning programme, and stress the importance of self-awareness in the learning process.

There are 24 teaching weeks, each with one full class lecture (80 students), smaller group tutorial session (12 students) of one hour and the availability of a 'drop-in' help session.

Each week normally addresses one study unit. Each study unit is divided into Fundamentals, Amplification and Study Notes. Currently this material is provided as 'hard-copy' and is also available through the VLE. The answers to questions are normally only available through the VLE to encourage wider engagement with the material.

The Fundamentals section includes basic questions in a progressive study sequence, together with key 'theory' statements and essential equations. This section is useful as a 'revision' programme for students who have met the material before, but it also provides the framework for the topic introduction in the lecture.

The Amplification section includes computer-based skills (mainly using EXCEL) that the student needs to master in the tutorials and/or further questions and applications relating to the topic. The Study Notes provide a more traditional 'text-book' coverage of the topic.

Summative assessment of the module includes two computer-based tests for mathematical and statistical skills and two portfolios of various tasks. The portfolio tasks include analysis of experimental results, writing a report following an information search and recording of performance in relation to the learning objectives. A modelling assignment using EXCEL also provides a combination of challenges that integrates the mathematical, statistical and IT sections of the course.

Self-assessment tests are provided through the VLE after five and ten weeks in each of the two semesters. The fact that the students take the test contributes to their module mark (through their progress record within the portfolio) but their actual score on the tests does not.

Pre-requisite Knowledge

The minimum entry requirement for all programmes taking this module, is Grade C in GCSE or equivalent in mathematics. In practice, a number of students have recently passed at A2 in GCE Mathematics and/or Statistics, but for a significant proportion, their mathematics background is mainly Grade C in GCSE taken at least two years previously.

This particular module is aimed at first year science students. Level of study for this particular module is first year science students. However, a second year module extends study in both statistics and experiment design, with VLE support material for both years.

How Are Students With Different Mathematical Backgrounds Supported?

The diversity of previous experience was the major consideration in the design of materials and approach. One aspect of the strategy uses the Fundamentals sections which

- Makes learning objectives very explicit (through example problems) – allowing students to focus their own learning requirements.
- Presents essential facts and equations – allowing ‘experienced’ students to confirm or revise their knowledge, and highlighting stepping stones for ‘new’ students.

Another aspect is to encourage student-centred learning through the provision of constructive feedback with an Induction Test and four Self-Assessment tests, worked examples and VLE support available on and off campus. Sufficient tutorial time was also available for struggling students to seek help – but see ‘Barriers’.

What Support Was Needed?

No special training is required. The delivery of tutorials in MS EXCEL uses computer laboratories equipped with data projection facilities for class demonstrations. Minitab is also used to a limited extent as it is already available on the system, but it is not essential.

The Barriers

The two greatest barriers are the diversity of student backgrounds, and the fact that many first year science students find it hard to develop interest in, and motivation for, mathematics and statistics.

The delivery structure (see ‘Different Backgrounds’) has relieved many of the ‘diversity’ problems. The ‘interest and motivation’ problem now becomes the next major issue. Lecture style, together with the use of relevant examples, appears to be very significant in addressing group attitudes. However, there still remains a small but difficult group of, usually weaker, students, whose lack of motivation and interest means that they still fail to engage fully with the available opportunities.

The Enablers

The most important enabler is the student-centred structure and mix of learning provisions:- hardcopy notes and questions supported by VLE software and self-assessment, together with lectures and tutorials.

The use of the Fundamentals sections as the focus for lectures has led to more a positive reaction from the students as well as a greater sense of student engagement.

Evidence of Success

Feedback from students through questionnaires and informal discussions shows that they are generally happy with the teaching style, and that each of the different aspects of learning provision are useful to significant subgroups within the total cohort. Performance records also show that students with limited mathematics experience can do well on the module.

How Can Other Academics Reproduce This?

This form of teaching is mainly applicable to topics where it is possible to define learning outcomes in some detail, hence its use for first year support mathematics and statistics. It is important that the structure of the learning resources is very clear to the students, that the style of the ‘lecture’ integrates with this structure and is not a stand-alone ‘traditional’ lecture.

The application of the mathematics and statistics to support different disciplines would require some restructuring of materials, particularly in respect of topics chosen and examples used.

Quality Assurance

Module curriculum, assessment details and teaching methods form part of the Module Specification, which must be approved through the Quality Assurance system in the university. Student representatives report to regular Programme Management meetings and students provide written feedback at the end of the module.

Other Recommendations

The fact that the staff both have backgrounds in science appears to be helpful in developing an approach which is sympathetic to the science students’ perceptions of mathematics and statistics.

Current developments include identifying key self-assessment questions that students should be capable of answering before starting each new study unit – important for progression in topics such as statistics and algebra.

Interactive Lectures

A Game Show Format for First Year Problem Classes in Mathematical Modelling

Colin Thomas ▪ School of Engineering ▪ University of Birmingham

Abstract

Problem classes are traditionally used in the teaching of mathematics. For a first year Chemical Engineering course in mathematical modelling, a quiz based on the TV programme "Who Wants to Be a Millionaire?" has been introduced, in a problem class supporting lectures. Following group work, with one set problem per group, students present their solutions to the rest of the class. The quiz follows the presentations. Each group is represented by a volunteer, who attempts to win chocolate prizes. The questions are both general, and specific to the particular problem done by the group. Besides reinforcing earlier learning, the quiz is fun. Certainly it appears to have been appreciated by two successive student cohorts. The lecturer and postgraduate demonstrator have also enjoyed the problem classes more than traditional formats.

Level of Material: First Year

The Execution

This modelling sub-module consists of 13 lectures, and 2 x 2h problem classes. A highly structured approach to modelling is taught in the lectures, with a strong emphasis on the key issue: how does one begin? Students usually struggle to turn a problem expressed in words into mathematical equations, and the method given to them is almost formulaic. Many examples of modelling are presented. These are taken from chemical and biochemical engineering practice and there are some of broader interest e.g. mathematical ecology. The engineering context is never allowed to overwhelm the modelling, and is often simplified and always explained. Detailed notes are provided in the lectures, and from next year it is intended to post these on the intranet. A-level mathematics is assumed, as is attendance at a prerequisite Engineering mathematics course in which calculus is revised. Mathematical modelling leads naturally to the derivation of differential equations. The methods of solving first order and some second order ordinary differential equations (ODEs) are taught.

The problem classes are intended to provide practice at modelling and in solving the resulting equations. The problem classes occur on (separate) afternoons in the second term. Both involve 25 to 30 students, and each is repeated, as the whole cohort is 55 to 60 students. The first problem class emphasises the development of models, beginning with the students criticising an attempt at "modelling" from a Tom and Jerry cartoon (a short video clip from which is used as light relief in an earlier lecture). The second problem class begins with more difficult model building involving second order ODEs and the quiz finishes this problem class.

In order to break up cliques and promote better interpersonal skills (i.e. team work), the students are allocated at random to one of 4 groups as they arrive. There are 7 to 8 students per group. The first 50 min of the second problem class is group work on one of 4 set problems, resulting in the preparation of a few overhead transparencies explaining the group's problem and its solution. The author and a postgraduate demonstrator circulate around the groups giving advice. Strong students are asked to help weaker students, so that all group members understand the group solution (and more interpersonal skills are practised). After a short break, one student volunteer from each group presents to the whole class (4 x 5 min). Originally it was conceived that any student might be asked to present, but compulsion was replaced by the offer of a reward; the volunteer receives a large chocolate bar as a reward. It is certainly the case that presenting complex mathematical derivations on a few overheads and in a few minutes makes a student have some sympathy with the lecturer!

After the presentations, the "Who Wants to be a Modell…er?" quiz begins. A second volunteer from each group faces 6 questions related to his or her group's problem. Four answers are given with every question. These are shown on an overhead projector to the student (who comes to the front of the room) and the rest of the class. Typically one answer will be correct, one will be obviously wrong, and the other two contain typical student errors. The student must choose one. As in the TV show the student has "lifelines". "Phone a friend " becomes ask a specific group member, "Ask the audience" means ask the class, and "50:50" works by the author removing two answers (not necessarily at random; a struggling student might be left with the correct answer and an obviously wrong one, whilst a stronger student might be left to choose between the correct answer and a common trap). The rewards for correct responses are chocolate bars, of the small party pack variety, for later distribution to the group. Unlike the TV show, one cannot lose everything: 1 correct answer overall is rewarded with 1 bar; 2 answers correct, 2 bars; 3 correct, 4 bars; up to 6 correct answers, 32 bars! The first question is always a joke (e.g. "Which of the following is a great model? Prof. Thomas, Kate Moss, a Skoda, La-La"; cheesy but gets things going in a non-threatening way!).

The questions are such that it is hard not to get at least 5 correct. When appropriate, discussion of the incorrect choices and the traps follows a question, using a whiteboard for notes when necessary. The 4 quizzes take about 40 min, completing a 2 hour class.

Pre-requisite Knowledge

The participants are first year Chemical Engineering students, all with Mathematics A-level or equivalent, generally at grade C or higher. Some students have grade A and/or A-level Further Mathematics. All have taken the first term of an Engineering mathematics course, which both revises and extends A-level knowledge. The modelling course runs simultaneously with the second term of this mathematics course.

How Are Students With Different Mathematical Backgrounds Supported?

The students are supported within their groups, with limited further support from the author and a demonstrator.

What Support Was Needed?

Both author and demonstrator have attended courses on small group teaching.

The Barriers

Volunteers are usually difficult to find, but chocolate seems to be sufficiently attractive. The volunteers are usually the better/more confident students, but the quiz is sufficiently stimulating to keep everyone's attention. If the groups were smaller, and there was a facilitator added to each group, no student could "hide". However, this would require much more demonstrating effort. If every group presented a solution, the problem class would become too long. The chocolate costs about £10 –15.

The Enablers

The problem classes mix several elements; this and of course the quiz itself keep student interest high. Chocolate is an acceptable reward and is key to the exercise! All the proceedings are very informal and essentially non-threatening, which helps improve participation.

Evidence of Success

Attendance at the problem classes is nearly 100%, and rarely does anyone leave before the end. Students have sometimes asked if they can attend the repeat of a problem class they have attended! The interactions between students, and between them and staff, are at a high level throughout. Student feedback is positive (leaving aside the obvious "What was best about the course?", "The chocolate."). The quality of answers to the assessed coursework (which follows the problem classes) has improved significantly. The author and demonstrator have both enjoyed the problem classes, more than for previous traditional approaches.

How Can Other Academics Reproduce This?

The methods are simple and should be easily reproduced by anyone who has seen the TV show. The game show (quiz) format could be applied in other mathematical problem classes, or problem classes in many subject areas.

Quality Assurance

Peer observation and monitoring of student feedback. Further issues can be raised at Staff/Student Committee.

Other Recommendations

The author intends to spend yet more time watching TV in the hope of finding other game formats that might be used in learning and teaching.

Interactive Lectures

New Approaches to Teaching and Learning in Engineering at the University of Strathclyde

Geoff McKay ▪ Department of Mathematics ▪ University of Strathclyde

Abstract

Mathematics is perceived as a difficult subject within engineering or science degree courses. Traditional lectures, with students passively listening to the lecturer or transcribing notes, do little for the students' image of mathematics. This style of lecturing and its lack of feedback can also be very unsatisfactory for the lecturer.

In an attempt to overcome these problems, an element of interaction has been introduced into lectures. This has been achieved by encouraging communication via the Personal Response System, part of the Interactive Classroom developed by the Department of Mechanical Engineering, University of Strathclyde. Lectures are problem-based, with students immediately given the opportunity to put the methods they have learnt into practice.

Level of Material: First Year (Scottish)

The Execution

Traditionally, students on science or engineering degree courses struggle with the mathematical element of their curriculum. Mathematics is perceived as a very *dry* subject, based on incomprehensible theory and applied to complicated problems. Too early in their degree courses students become de-motivated where mathematics is concerned. Attendance levels drop, students are unwilling to work through questions on their own and as a result performance levels suffer.

In order to counter this, the Department of Mathematics has successfully undertaken an overhaul of its service teaching provision. One element of this overhaul is the use of *theory notes* in all first year classes. In reality these notes contain little theory but cover in detail the methods introduced in the class. Lectures are no longer an exercise in dictation. (Many students struggle with dictation simply because of the teaching methods they are accustomed to from school or college.) Students are given concise, accurate and focussed versions of the background material for each section of the class. They are not expected to copy theory from the board. Theory and methods are covered in a shorter period in class and consequently lecturers now have more time to demonstrate the important concepts via a series of relevant examples. By following these illustrations as they are developed in lectures (in conjunction with the clear theory notes), students are in a better position to attempt problems successfully on their own. The students respond well to this type of examples-based teaching and their understanding has improved.

Another element of the teaching overhaul was the introduction of Class Tests. These Tests have proved highly successful in a number of ways. They act as an incentive to students to maintain their work level, thereby gaining an exemption from a longer end of semester exam. They also provide feedback to the student and lecturer on performance as the class progresses.

Whilst examples-based teaching has certainly proved successful, the student still learns *passively*. All too easily the students leave the room but forget what they have learnt because they have not put it into practice. Rather than rely on the student making the effort outwith the lecture, it would be preferable for the student to attempt examples *during the lecture itself.*

Such a combination of lecture and tutorial has been introduced using a classroom communication system, the Personal Response System (PRS). Students become familiar with PRS early in the Mechanical Engineering course at Strathclyde, and employ it in many engineering classes. This has been extended to include Mathematics in first year. Students are allocated a PRS handset with a unique ID identifier. They are also assigned to groups of four students that they remain in throughout the semester.

During lectures, students are asked to work through examples based on material recently covered by the lecturer. They are encouraged to do this within their groups, thereby using fellow group members as a learning resource. Students are also given a number of possible answers to the problem, five choices, say. The PRS handsets are used to transmit the students' responses and confidence levels to the lecturer via receivers and PC software. The overall class performance can be displayed graphically and discussed with the class. (The obvious analogy is with the *"Ask the Audience"* section of a current, highly popular quiz show!) A similar approach is adopted in tutorials, although there the students are given a series of problems to work through and are encouraged to converse with individual tutors.

The choice of question set to the class is highly important. Sometimes this might be a snap question where an instant response is required; students are encouraged to recognise solutions or the appropriate approach without the need to resort to pen and paper or a calculator. Questions may also be broken up into smaller stages; the students can be asked to respond at each stage of the process. The use of distracters amongst the offered solutions is also very important. Every year students make the same simple mistakes. By providing possible answers based on typical mistakes, the lecturer can demonstrate errors common amongst students and help eliminate them.

To demonstrate this, the following are two simple questions used in class during the Differentiation Block. The first is a snap question where the student is expected to respond quickly. For the second question the student is allowed more time.

1. Differentiate cos(5x) with respect to x.

   Possible responses: -sin(5x), (-sin(5x))*5, (-sin(x))*5, -sin(5x*5), sin(5x)*5.

2. If $x(t)$=cos(t) and $y(t)$=sin(t), then calculate d^2y/dx^2.

   Possible responses: -cosec^3(t), -cosec^2(t), -cosec(t), -cos^2(t), -cos(t).

Other course materials may be found at **http://www.maths.strath.ac.uk/coursemats/652/index.html**.

The benefits of the PRS system are many-fold:

- It promotes student activity/discussion within a class.
- It provides immediate feedback to the students and helps them gauge their performance within the class. Assignments have also been constructed for the students based on multiple choice answers. Again, selections can be transmitted to the lecturer using handsets and immediate feedback given to the student.
- It also provides an immediate response for the lecturer. More time can be devoted to a concept when the students perform badly. Alternatively, by storing the responses from each lecture in a file, the lecturer can analyse a student's performance throughout the whole year. Class attendance can also be monitored in a similar fashion.
- Students are more confident about mathematics, and methods have become more familiar.
- By interrupting the elongated teaching slot, the questions set in class and subsequent discussions help maintain student interest and concentration.

Pre-requisite Knowledge

As a first year class, the students are expected to have no pre-requisite knowledge other than the appropriate entry qualifications in mathematics. However, the students examined here also benefit from being exposed to the PRS handsets and peer instruction at an early stage.

How Are Students With Different Mathematical Backgrounds Supported?

All students have attained an appropriate level before entering University and the class is designed with this level in mind. Naturally, students who do have higher qualifications find the transition into University easier as much of the material may be revision. However, students without these higher qualifications are not disadvantaged in any way.

What Support Was Needed?

Interactive teaching at Strathclyde was developed by the Department of Mechanical Engineering (within the NATALIE project, New Approaches to Teaching And Learning In Engineering). Several members of staff in that Department provided support with training in the relevant software, hardware provision and timetable re-organisation. Mechanical Engineering also carries out much of the administration (e.g. initial PRS and group allocation).

Providing lecture notes for all students in service teaching classes is an expensive undertaking. There is considerable staff-effort required in producing high quality notes, exercises, solutions, lecture slides, relevant examples, etc. Similarly, the financial implications of photocopying many copies of theory notes are considerable.

The Barriers

Following several years experimenting with PRS handsets in tutorials, the Department of Mathematics made the decision to employ this type of teaching in lectures. However, the major barrier to this was organisational. The recently refurbished NATALIE rooms at Strathclyde are in high demand. The course in Mechanical Engineering is also extremely popular. It proved difficult to timetable the students in the appropriate rooms. Rooms were either in use or not large enough for the Mechanical Engineering cohort. These restrictions may also prove significant in determining whether we can extend the methods to other cohorts of students.

Thankfully, we have not had to overcome any resistance to innovative teaching methods within the University, although passive learning still dominates in too many areas of teaching!

The Enablers

Students were encouraged at a very early stage to discuss their work within groups or with the lecturer. The confidence gained by successfully attempting problems in the lecture, and the instant feedback provided by the lecturer, helps the learning process.

Evidence of Success

As part of the NATALIE project, education specialists at Strathclyde have interviewed students. Their views on this type of teaching (in mathematics and engineering) have been extremely positive. Questionnaires completed by the students also provide positive responses. (Interestingly, many students requested that even more use be made of the handsets in mathematics lectures.) Overall, the performance of the cohort examined here (in Class Tests or Exams) is extremely high compared with students studying the same material but not exposed to the interactive teaching approach. (It should be pointed out, however, that Mechanical Engineering has higher entry requirements than most courses within the Engineering Faculty at Strathclyde.)

How Can Other Academics Reproduce This?

Theory notes with examples carried out by the lecturer have proved very successful at Strathclyde for a number of mathematics classes, even without student interaction. However, using the PRS allows the lecturer to involve the student more actively in the learning process.

The approach described here is essentially a mix of tutorial and lecture. It could be employed in a variety of mathematics classes, although perhaps it is more suited to lower level material or problem-based subjects (e.g. mechanics). However, without some form of student monitoring it may prove unsatisfactory. Without a response system some students will inevitably "hide" (especially if student numbers are large), choosing to put in no effort in class. The PRS system, whilst providing feedback and although not foolproof, is a valuable tool for overseeing student effort and performance.

References

EduCue (PRS distributor): http://www.educue.com

NATALIE Project, University of Strathclyde:
http://www.mecheng.strath.ac.uk/natalie.asp
http://www.ltsneng.ac.uk/nef/features/featurearchive/natalie.asp

Improving Student Learning through Collaboration

Developing the Interface between Engineering and Mathematics at Edinburgh University

Interview with John Christy ▪ Chemical Engineering ▪ University of Edinburgh

Abstract

A few years ago the department of mathematics at Edinburgh looked at the problem of interfacing between mathematics and engineering courses and came up with a system to enable greater student understanding. The implementation of this system and how it is helping students is reviewed in this case study.

Level of Material: First and Second Year

The Execution

It is important that all students reach the stage of being able to handle complex differential equations. They need to understand the meaning of the various terms of the equation in order to decide from physical situations whether particular terms are zero because of various symmetry concerns or indeed, whether the term ought to be negligible. To be able to understand these concepts well it is important that the students have a good grounding in mathematics. A few years ago the mathematics department at Edinburgh looked at the problems of interfacing between mathematics and engineering courses and came up with a system to enable greater student understanding. Now all first and second years are taught mathematics by the mathematics department, while using examples and problems are provided by the engineering department.

The engineers supply copies of notes from, for example, fluid mechanics lectures including all the tutorials and tutorial solutions. This enables the mathematics lecturers to see the ways in which the mathematics they are teaching is going to be used later on in the course. The mathematics lecturers then select one or two examples, which are presented during their own tutorials and lecture courses. The objective is to teach first and second year students in terms of examples that are relevant to engineering to help them see the problems within an engineering context.

The comments from the students have been positive. Difficulties still exist but there are fewer complaints based on the fact that the students are seeing the relevance of what they are doing with the mathematics. This is viewed as the first step towards motivating the students to take mathematics more seriously.

Pre-requisite Knowledge

The engineering tutors are careful in terms of the selection of students to their courses. They often ensure the level of mathematics of students is appropriate before offering them a place, and this is done in consultation with the mathematics department.

How Are Students With Different Mathematical Backgrounds Supported?

The mathematics department has two first year courses. One course is taken by the majority of students to give them a standard background in mathematics. The other course operates at a more remedial level for students with a C in Higher Grade mathematics, D in A level mathematics or other qualification. Numbers on this course are limited to 50 students.

The main course gives some examples and goes beyond what is needed for engineering. The remedial course sticks very much to what the students actually need before they progress to second year mathematics.

The Barriers

The biggest barrier has been getting from the issue of "what mathematics it would be nice for students to know" to "what mathematics do students need to know to do the engineering". Whether the student can do the mathematics from first principles or not is neither here nor there in terms of engineering practice. But there is still a debate as to whether students can really use something as a tool without understanding where it has come from. The mathematics department is looking at the ways engineers use mathematics, and has tailored a number of these at first year level.

There will always be a debate over what students need to know. Some of what we have assumed the students need to know has ultimately been a barrier to their progression. Students make statements like "I could cope with this at school but now having been taught this at university I am no longer even confident of what I was able to do at school." At school they had used rules, e.g. for differentiation, but once at university the mathematicians went back to first principles and so the students panicked over their ability to use these tools from that point onwards.

The Enablers

The main enabler has to be the close liaison between the engineers and the mathematics department. Three or four years ago there was increasing evidence that first and second year chemical engineering students were having difficulties with the mathematics, and consequently the engineering department was facing a growing lack of interest and understanding.

The mathematics department had introduced "engineering examples" within the mathematics teaching. However, the questions tended to reiterate the mathematician's point of view and in many cases, the content was proving too advanced for the students. It was becoming vital that the mathematics department explored engineering approaches and ways to teach mathematics from an engineering viewpoint.

The Mathematics – Engineering Liaison Committee received funding to employ a facilitator to look at the interface between the mathematics and engineering departments. The role of the facilitator was to make the mathematics department more aware of the links between the mathematics being studied and the practical engineering.

How Can Other Academics Reproduce This?

Different universities are taking different views on the teaching of mathematics. Some engineering departments have decided to take on the mathematics training themselves. Others have gone to a halfway house, taking on board certain things themselves where the use of the mathematics is only ever going to be used in a fairly clear engineering context. Edinburgh has actually gone down the route of using their mathematics department and what is interesting is that the mathematics department have come up with a solution themselves. They employed someone with an engineering background to liaise between the two departments, to develop an understanding of what the chemical engineers were going to do in their course.

It has also relied upon the trust of the engineers – when solutions are going to be scrutinised by mathematicians it is rather daunting. But this is the only way forward.

Using Technology for Teaching Engineering and Science

Graphics Calculators

Using the Graphics Calculator to Support Mathematics for Engineering Students

Interview with Susan Jackman and Ann Evans ▪ Centre for Mathematics and Statistics ▪ Napier University

Abstract

For first and second year engineering students at Napier University, the TI-83 graphics calculator plays a major role in an integrated technological approach to mathematics. This case study reviews the process of integration and its current position in the teaching of students.

Level of Material: First and Second Year

The Execution

The embedding of calculator technology into the curriculum commenced in 1994.

32 students in an electrical engineering class were asked to buy a Texas Instrument TI-82 graphics calculator. Their lectures and tutorials were redesigned so that the calculator could be fully integrated into the teaching process. The aims of the project included promoting conceptual understanding, increasing students' confidence about mathematics and improving the accuracy of their work [1].

A flexible resource was created by connecting the lecturer's calculator to a viewscreen, allowing the lecturer to teach within an ordinary classroom with an overhead projector (OHP). This enabled students to see the display and to follow the steps on their own calculators.

Topics taught during the pilot study included functions, vectors, matrices and determinants, complex numbers, calculus and statistics. Examination questions were reviewed to take account of calculator use and it was made clear to students how much 'working' they must include in their written answer. This was important so that logical thought and the communication of mathematics were encouraged.

The pilot study continued with the equivalent class in 1995. Structured interviews were held with the group using the TI-82 and with a control group studying the same module without the use of a graphics calculator. Students were asked questions which measured their understanding of important concepts. It was found that significantly more of the class who had used the TI-82 demonstrated a better understanding of simultaneous equations, exponential functions, derivatives and stationary points than was the case with the control group [1].

Upon gaining approval from the engineering department, in 1997 the new TI-83 became embedded within the curriculum. The use of the technology was extended to include students on Mechanical Engineering and Science degrees. It became an integral part of the teaching of mathematics to engineering students at Napier University. Today, all first and second year students taking Engineering Maths Modules are encouraged to buy the TI-83 and they are each provided with a set of notes about the calculator.

In the lectures, teaching mathematics is based around four stages: explaining the theory to students; worked examples; examples with the aid of the graphics calculator; and illustrating the maths topics in a context relevant to engineering. In the tutorials, the calculator is used in exercises designed to develop competence in mathematical techniques, and in some cases to develop skills toward mathematical modelling. In both the lectures and the tutorials the calculator provides a visual understanding of graphical solutions and the related arithmetic and plays an integral part in the teaching of mathematics in engineering applications. Generally it enhances the teaching and learning of mathematics and consolidates the coursework.

Pre-requisite Knowledge

First and second year students have a wide variety of mathematical backgrounds. Many students enter first year courses with disappointing maths results; some will have struggled throughout their secondary education and some are mature students re-entering education. Some possess few mathematical skills, for example in basic manipulative algebra and the rules governing mathematical processes.

Students do however have IT skills and in many cases are familiar with graphics calculators. Academics teaching the engineering mathematics modules assume that students have limited mathematical knowledge. The aims of the modules are to break down barriers and help students gain confidence. The calculator is just one feature in that process.

How Are Students With Different Mathematical Backgrounds Supported?

It is important to recognise that the calculator is part of a teaching and support package, which also includes: MathPlus – a drop-in centre, computer based assessment, flexible learning booklets, tutorial exercises and applications, use of mathematical software, staff workshops and module meetings. During class the first year students spend time revising foundation maths and are encouraged to use the calculator throughout the engineering course.

What Support Was Needed?

The calculator is an integral part of today's education system and almost all students have encountered the tool. For those who have problems with using the calculator, assistance is offered at the beginning of the academic year and throughout. An arrangement has been made with Waterstone's in Edinburgh for the students to buy the TI-83 graphics calculator at a reduced price. In return for the barcode the students are issued with a free copy of "Introduction to the TI-83 Calculator".

At the beginning of each semester workshops are run on the graphics calculator for the staff. They are shown the TI viewscreens, which enlarge and project the image of the graphics calculator display so it can be viewed by an entire group of students. During the semester there are regular meetings/discussions with all academics involved in the engineering maths modules.

The Barriers

The main barrier is the students' attitude – many lack confidence. They have had negative experiences in maths and seriously lack study skills. Some students complain about having to purchase the calculator, but most appreciate the value of buying one, as it will be used throughout the four years of the course. In special cases the department will supply one, to be returned once the module is completed.

The Enablers

There is no major difficulty in using the calculator. These days students have the necessary skills to use the technology and its use provides an incentive to learn. Furthermore, when incorporated within the teaching, a calculator such as the TI-83 enables the students to see the maths topics in a context relevant to engineering. It helps them to visualise mathematical concepts. Such IT products are changing the way mathematics is taught and are having a significant impact on the curriculum.

Evidence of Success

Students fill in a standard questionnaire supplied by the faculty. They are asked questions about the various aspects of the module, including what they liked most about the modules and what they liked least. These responses are summarised and circulated to the relevant members of staff and the School of Engineering.

In a recent module questionnaire response, general comments were extremely positive and related to the teaching methods including the use of the calculators. One student stated, "I feel the teaching methods are excellent…" another response was, "The way the class is taught helps me learn more effectively". There were no specific comments relating to the calculator other than one student complaining about the cost of purchase.

How Can Other Academics Reproduce This?

In general the introduction of the calculator raises new issues, as there is a need to change the style and approach to teaching. It is important that academics become familiar with the technology so they can demonstrate to a class of students. Judgments must be made about how much time should be allocated to the use of the calculator. In addition, appropriate types of investigative work must be identified. The financial implications must also be considered. It is vital that all students have the same model of calculator. Students were not required to buy a first year mathematics textbook, and they recognised that the calculator would be useful throughout the four years of the course.

Quality Assurance

All maths modules are validated by the University's Quality Enhancement Services (QES) and any changes have to be approved by a Faculty committee before they are presented to QES. The marking of the assessments is shared so that continuity and consistency are achieved. There are module meetings before and throughout the semester to discuss quality in reference to Napier's LTA strategy.

Reference

[1] Four Years Experience of Graphic Calculators in the Mathematics Classroom, *Mathematics Today*, 35(1), Scott, T., Jackman, S., Ballantine, H., and Harvey, J., (1999).

Graphics Calculators

Teaching Mathematics to "Science with Management" Students Using A Graphic Calculator

Interview with Diana Mackie ▪ Centre for Mathematics and Statistics ▪ Napier University

Abstract

The introduction of the graphics calculator has provided the fourth year students taking Science with Management Studies with an interactive learning tool. This case study reviews its introduction into the course Discrete and Continuous Models at Napier University.

Level of Material: Fourth Year

The Execution

The module Discrete and Continuous Models is offered to fourth year students enrolled in a BSc Science with Management Studies. The objectives of the course are to formulate and solve models involving differential and difference equations for a range of applications in physical sciences, biology and management, and to investigate the behaviour of solutions of discrete and continuous models.

During the last five years, the content of the course has remained the same. Any changes have been brought about by the introduction of technology. Initially a software package called Nodes was used to help the students produce graphical solutions. Later this was replaced with the TI-86 graphics calculator and the computer package Mathwise.

Throughout this period the emphasis of the course began to shift towards a combination of traditional mathematics and graphical output, to assist the investigation and interpretation of behaviour models. The alternative approach to learning, teaching and assessment presented by the technology began to motivate the students and encourage interaction.

Today the calculators are provided to the students during the course. They are used to plot solutions of differential equations, to construct phase plots and to explore solutions of discrete models. These tools enable the students to produce answers more easily and in a different form. Mathwise is used for revision and reinforcement. The overall assessment of the course also changed to an examination in which the calculator is used (70%) and a computer/calculator based assignment (30%). The assignment involves the investigation of a model given by a pair of differential equations in which the TI-86 is used to produce a phase portrait of a particular case. The results are interpreted and discussed in a report. The 13-week teaching programme is a combination of lectures, calculator workshops which provide an opportunity to practise basic techniques, Mathwise labs and tutorials.

In early lectures the students revise the solution methods for first and second order equations. Linear and non-linear models are also introduced. Students are familiar with this material but little time has been spent on looking at models in depth. The next stage of the programme looks at systems of differential equations and the construction of phase portraits using a graphics calculator or a computer package. There is also an in depth study of models from physical and social science e.g. predator-prey, competitive species and epidemics (see Model – Epidemics question). The final part of the course looks at difference equations.

For the academic the calculator is used in the teaching process, illustrating changes within the models as they occur. The TI viewscreen can be used to enlarge the image from the graphics calculator and display it on an OHP screen. Throughout there is an ongoing process of relating the graphical output to the analytical solution and the traditional mathematics.

For the students, carrying out the coursework requires a genuine understanding of the mathematics and the procedures involved. The calculator provides the means to create a graphical representation of the phase portrait. It is a learning tool, which enables both the academics in the teaching environment and the students to achieve something that could not be achieved without it. It also assists the students to learn something more effectively and efficiently than they could otherwise [1].

Pre-requisite Knowledge

The class presents varying levels of attainment. For some their overall mathematical background is limited, for others a lack of recent study of differential equations can prove a problem.

How Are Students With Different Mathematical Backgrounds Supported?

The class is small, therefore there can be ongoing individual support. The students use Mathwise modules to revise differential equations.

The Barriers

Approval of the external examiners had to be obtained to include the calculator in the course structure. They are now supportive. There are no major problems in using the calculator in the teaching process or as part of the module assessment and the examination.

The Enablers

The calculators are provided on loan and each student is given a manual.

Evidence of Success

The feedback from the students is extremely positive. The students enjoy the work and put in a lot of effort. The assessment work is detailed and indicates a high level of understanding.

How Can Other Academics Reproduce This?

- To integrate the calculator within a module academics must have a clear understanding of the role it will play.
- Approval is required if the technology is to be used as part of the coursework, assessment and also in the examination.
- The coursework must be relevant and interesting. There must be a strategy in obtaining the calculators and making them available to the students.
- Class time must be allocated to use the calculator, e.g. calculator workshops.

Quality Assurance

All modules undergo an annual review where feedback on teaching and assessment is examined. Comments back from the students are extremely favourable.

Reference

[1] The Use of IT to Support Mathematics for Science and Engineering, http://ltsn.mathstore.ac.uk/workshops/maths-support/Mackie.pdf, Mackie, D., (2002).

MA40007 Tutorial

Model: Epidemics

1. Solve the epidemic model:

$$\frac{dS}{dt} = -rSI$$

$$\frac{dI}{dt} = rSI - \gamma I$$

on your graphics calculator, given r = 0.1 and γ = 6 for the following cases:

$S_0 = 100$; $I_0 = 5, 10, 40$

$S_0 = 80$; $I_0 = 5, 10, 40$

$S_0 = 60$; $I_0 = 20$

$S_0 = 40$; $I_0 = 10$

a) In each case, (i) plot the phase path for $0 \le t \le 5$;

(ii) use the TRACE facility on your calculator to estimate

- The maximum number of people infected at any one time;
- S_∞;
- The duration of the epidemic;

(Assume the epidemic has ceased when $I < 0.1$)

(iii) deduce the total number of people who catch the disease as a percentage of the population.

b) Construct a phase portrait and explain the behaviour of the model.

Reference: Modelling with Differential Equations, Ellis Horwood, Burghes, D. N., and Borrie, M. S., (1982).

Spreadsheets

Using Spreadsheets to Teach Quantum Theory to Students with Weak Calculus Backgrounds

Kieran Lim ▪ School of Biological and Chemical Sciences ▪ Deakin University ▪ Australia

Abstract

Quantum theory is a key part of the chemical and physical sciences. Traditionally, the teaching of quantum theory has relied heavily on the use of calculus to solve the Schrödinger equation for a limited number of special cases. This approach is not suitable for students who are weak in mathematics, for example, many students who are majoring in biochemistry, biological sciences, etc. This case study describes an approach based on approximate numerical solutions and graphical descriptions of the Schrödinger equation to develop a qualitative appreciation of quantum mechanics in an Australian University.

Level of Material: Second Year

The Execution

The aim here is to teach the qualitative results that arise from applying mathematics to physical and chemical systems, but without the mathematical rigour: "teaching maths without the maths". The "new calculus" advocates the "rule of four" (numerical, graphical, symbolic and verbal descriptions) to deepen students' conceptual understanding. Students who have a weak background in mathematics do not have the knowledge of calculus required for the usual symbolic algebra approach to quantum theory. This case study illustrates how a combination of numerical, graphical and verbal descriptions can be used to overcome the lack of symbolic knowledge or ability.

Quantum theory is a key part of the chemical and physical sciences. In preceding semesters, students have collected spectra that show atoms and molecules only absorb (and emit) certain photon energies which are characteristic of that atom or molecule: other photon energies are not absorbed. Students have also learnt from classes and textbooks that:

- "Allowed" energy levels are quantised, but usually without appreciation of why.
- Electrons and atoms exhibit both wave-like and particle-like behaviour.

The Schrödinger equation for an electron-in-a-box (the Kuhn model) is introduced as two coupled first order differential equations. The derivative is explained as the "slope of a function". The first order Euler method for generating numerical solutions is explained. No calculus is required as the Euler method can be derived from the definition of average slope. Students create their own spreadsheet (to find numerical solutions to the Schrödinger equation), or are supplied with a spreadsheet written by the lecturer.

The wavefunction solutions are classed as "valid" or "invalid" depending on whether the boundary conditions are satisfied as energy is varied (the shooting method). Students discover that energy determines the wavelength of the wavefunction, and that valid solutions require that only special ("allowed") wavelengths will fit the dimension of the box. Further exploration shows that energies decrease as the box is enlarged ("delocalisation lowers energy") and that energies increase as barriers are introduced (Kronig-Penney model).

The key results from the exploration activity are reinforced in class and generalised by considering other potential-energy functions, including functions (e.g. triangular-well potential) for which no analytical solution exists.

It is observed that the number of nodes (the zeroes or roots of the wavefunction) increases with energy and that the qualitative shape of the wavefunction can be generated from the nodal pattern. (One strategy in de Bono's Lateral Thinking is to concentrate on what is not present — i.e. the nodes or zeroes — in order to obtain what should be present — the wavefunction). Wavefunctions can then be generated from nodal patterns in 2-dimensions and 3-dimensions. For example, the rotational wavefunctions are generated by considering nodal patterns (and hence wavefunctions) on the surface of a sphere (the original Hamiltonian model for waves on a spherical ocean. Note that these "spherical waves" correspond to combinations of the spherical harmonic functions, and are obtained from the symmetry — "topology" — of the nodal patterns).

Pre-requisite Knowledge

The minimum mathematical knowledge required is to know about functions and to understand and manipulate the definition of average slope. All students entering university from secondary school (high school) will have this pre-requisite knowledge. Students also require some computer literacy and minimal knowledge of creating formulae in spreadsheets — see comments under "What Support Was Needed?"

How Are Students With Different Mathematical Backgrounds Supported?

This topic has been designed for students with minimal mathematical background. Students who have a stronger mathematical background can be extended by emphasising the symbolic approach — solve the Schrödinger equation using calculus. Extensions such as numerical integration to test orthogonality of wavefunctions are also possible.

What Support Was Needed?

The Information Technology Services Division runs computer-training sessions during Orientation Week at the start of the academic year. The university is investigating the implementation of on-line self-paced modules associated with the International Computer Driving Licence (ICDL), which includes training in the use of spreadsheets. The use of spreadsheets is demonstrated to students who are still not confident in their use at the start of this topic.

The Barriers

In some Australian states, it is possible to complete high-school mathematics without any calculus. At many Australian universities, students choose statistics as their first-year mathematics. Hence, traditional approaches to quantum theory are not practical.

Physical chemistry, and especially quantum theory, are viewed as difficult topics within the chemistry course because of the level of mathematics (second order PDEs, basis sets, etc.) required. This has been overcome by de-emphasising the traditional symbolic approach and instead focusing on the numerical, graphical and verbal descriptions of the topic.

The Enablers

The major enabler is the use of spreadsheet software to generate numerical solutions of differential equations. For mathematically weak students, spreadsheets are preferred over MathCAD, Mathematica and similar programs, because spreadsheets are perceived as being more “ordinary” and easier-to-use.

Most students have access to spreadsheet software at home, without incurring additional licensing cost for the student or the university. In other topics, spreadsheets are also used for data analysis and plotting, again in preference to more specialised programs like SPSS or Minitab, etc.

Evidence of Success

The evidence of success is that students who have no knowledge of calculus are able to complete this introductory topic.

How Can Other Academics Reproduce This?

Examples of spreadsheets and other documents are located at the unlinked URLs in the references below.

Similar approaches using numerical solutions can be used to teach the differential equations associated with chemical reaction kinetics.

Quality Assurance

Second and third-year subjects (units) at Deakin University undergo formal student feedback/evaluation at least biennially, with informal feedback/comment to teaching staff. Students who have no knowledge of calculus are able to complete this introductory topic.

Other Recommendations

The use of specialised software (such as Origin, SigmaPlot, MathCAD, Mathematica, SPSS, Minitab, etc.) should be deferred until students undertake research programs. Until then, generalist software (“worldware”) should be used across all disciplines to foster expertise in software that is in common usage in the general workforce outside universities. However, surveys of students at Deakin University indicate that they are less confident with the use of spreadsheets than other common generic software types, e.g. word processors, web browsers, electronic mail. Further, the skill level with each type of software is highly variable.

References

The approach described in this case study has been influenced by the American M.U.P.P.E.T. program and the “new calculus”:

“Student programming in the introductory physics course: M.U.P.P.E.T.” *American Journal of Physics*, 1993, 61, 222-232, Redish, E. F. and Wilson, J. M. (1993) http://www2.physics.umd.edu/~redish/Papers/mupajp.html

University of Maryland, Published papers describing *M.U.P.P.E.T.* http://www.physics.umd.edu/ripe/muppet/papers.html, Redish, E.F. (1995) (accessed 20 December 2001).

Preparing for a New Calculus, *Mathematical Association of America*, Washington (DC), vol. 36, Solow, A.E. (ed.) (1994).

“Introduction” to: Stewart, J., Calculus, 4th Edition, Brooks/Cole, Pacific Grove (CA), Stewart, J. (1999).

URLs

http://www.deakin.edu.au/~lim/teaching/quantum_mech/Asgnt_1.pdf

http://www.deakin.edu.au/~lim/teaching/quantum_mech/Asgnt_1_soln.pdf

http://www.deakin.edu.au/~lim/teaching/quantum_mech/Jillian.xls

http://www.deakin.edu.au/~lim/teaching/quantum_mech/Jillian_demo.xls

http://www.deakin.edu.au/~lim/teaching/quantum_mech/quantum_well.xls

http://www.deakin.edu.au/~lim/teaching/quantum_mech/Morse.pdf

http://www.deakin.edu.au/~lim/teaching/quantum_mech/Morse_exercise.XLS

Simulation of Linear and Non-linear Dynamic Systems using Spreadsheets

Malcolm Henry ▪ Department of Chemical Engineering ▪ University of Bradford

Abstract

EXCEL has been used to provide simulation facilities in support of teaching control to engineers. This dictates a sampled data approach which fits in naturally with digital implementation of control. The technique also allows students to explore the affects of non-linearities in systems such as control signal saturation. It provides a 'hands-on' dimension which students find valuable. The approach is capable of use with other dynamic systems and is not restricted to teaching control.

Level of Material: Third Year

The Execution

Control is a subject most students find difficult. Whilst the mathematics is not particularly onerous, the understanding required is demanding.

Years ago, when I was a student, we had analogue computers and these were a great help in understanding dynamic systems. One could see them at work and change parameters. Similar things are provided digitally today with packages such as MATLAB. However, such packages are not available on every machine, and particularly on students' own machines. Also, when a 'clever package' does something for you, you aren't quite sure how it did it. With a spreadsheet such as EXCEL, everyone has a copy and the students has to set up everything – there's nothing clever being done for him/her.

Pre-requisite Knowledge

Whilst students have met differential equations, few have understood what they are modelling. The approach described here is designed to develop that necessary understanding.

Some understanding of digital signal processing is developed at the same time, in particular the need to sample frequently enough.

How Are Students With Different Mathematical Backgrounds Supported?

The more able students get through tutorial problems quickly and/or elect to work in their own time asking for help as required. The less able students are therefore more fully supported in the tutorial time made available in the form of computer labs.

What Support Was Needed?

Staff availability, with a PC, in the form of 'face to face' contact is the only real way of resolving problems though small difficulties may be resolved at a distance using phone or email.

The Barriers

Some students devote more time to avoiding the difficulties than would be needed to resolve them! The idea that they need to 'get their heads round a problem' is considered strange, abhorrent and beyond their abilities – till you oblige them!

The Enablers

The lecturer – who used the technique to cover more ground more quickly, more thoroughly and with a greater level of understanding on the part of the students.

Evidence of Success

Better understanding – especially evident when students come to their final year projects. Whilst that's not an objective test, it tells me what I want to know.

Quality Assurance

Module descriptors and exams subject to the normal review procedures. Ultimately, though, it's a matter of letting a competent teacher get on with the job.

Other Recommendations

This technique is not limited to the teaching of control. Indeed, it could be used quite widely. For example, in chemistry and chemical engineering it is a good way of examining reaction dynamics.

Ultimately, why not use the technique from the beginning with differential equations. It avoids the situation where students learn how to solve differential equations without knowing what the solution means! (Yes! I have to admit that I was just such a student!)

Simulation of Closed Loop Systems

Now let's try some feedback

All we have to do to apply some feedback is to add another formula to the spreadsheet so that the input (control signal) depends on the error, the difference between the output, y, and the setpoint.

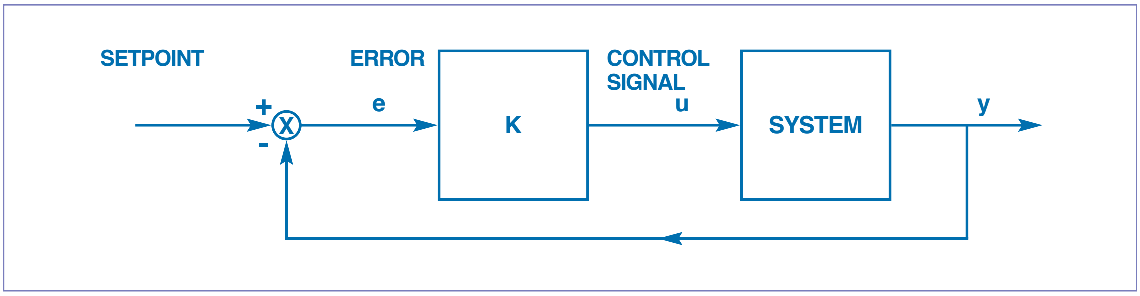


Figure 1: Block diagram for proportional control

The simplest formula would be:- $u_n = K(\text{setpoint} - y_n)$

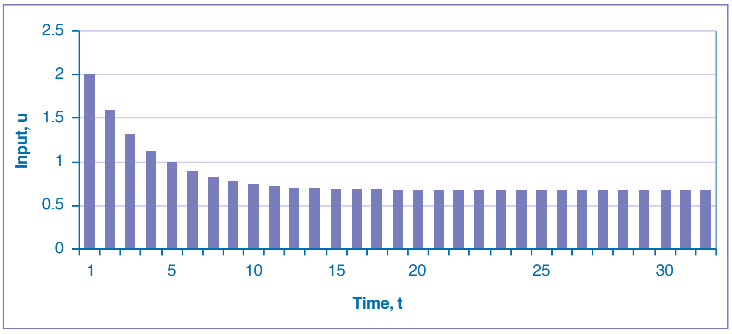
So now we have:- for setpoint = 1, K = 2

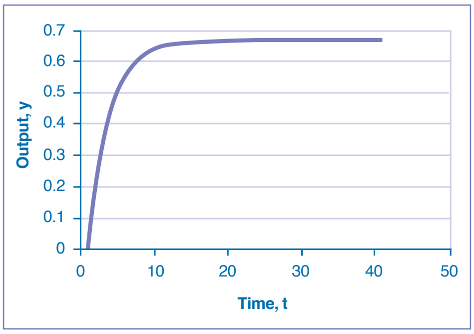
Time, t	Input, u	Output, y		
0	2	0		
T	1.6	0.2	y = 0.1 x 2 + 0.9 x 0	u = 2(1-0.2)
2T	1.32	0.34	y = 0.1 x 1.6 + 0.9 x 0.2	u = 2(1-0.34)
3T	1.124	0.438	……..	

Obviously the response is going to be very different from the open loop response. As with continuous control, it will be the same sort of shape, but on a shorter time-scale and with the response not settling at the setpoint.

By varying K we can observe (i) the varying steady state error
(ii) the varying speed of response
(iii) the changing control signal
and, because it's a discrete system, (iv) overshoot and eventual instability.

It is good practice to use separate plots for the control signal and the response. The former changes step-wise and should be represented by vertical bars.The latter is continuous and should be represented by a line:





We have done all this by programming two cells on the spreadsheet and calling up some displays. We have explored a number of things, hands on, in our own time.

A careful look at what we are doing

In the above simulation, the controlled input, u, only changes at a sampling interval. In between u is constant. That is why it plotted as a series of vertical bars. We could describe u as being 'piece-wise constant' or we could say it changes 'step-wise'. It is not a continuous function of time. However, the output, y, shown above, is a continuous signal and so it is plotted as a line.

What we are simulating is a sampled data control system where T is the sampling interval. This corresponds to what happens when we use a computer to control a process.

Specialist Software

Exploiting Synergies between Teaching Mathematics and Programming to Second Year Engineering Students

Jens-Dominik Müller ▪ School of Aeronautical Engineering ▪ Queen's University Belfast

Abstract

Students in Aeronautical Engineering are taught MATLAB in the first year to provide them with programming skills and for use in later courses. In the second year MATLAB is used to enhance the teaching of linear algebra and to apply the mathematical techniques to engineering problems.

Level of Material: Second Year

The Execution

The course links the learning of second year Mathematics, mainly linear algebra, and computing. The course is one module of 72 contact hours over the entire academic year. 36 hours of mathematics and 6 hours of computing are taught in the first semester, leaving 18 hours of mathematics and 12 of computing for the second. The bulk of the mathematics is taught in the first semester in order to provide the tools for a computing assignment in the second semester drawn from an aeronautical engineering problem in structures or aerodynamics.

The first semester mathematics teaching consists of, on average, two hours of lectures and one hour of in-class exercises. Four exercise sheets with solutions are handed to the students during the first semester. The computing in the first semester is kept very simple and parallel to the mathematics. Two problem sheets are set, asking the students to program some basic linear algebra techniques they are familiar with from their hand-calculations, e.g. calculate a matrix-matrix product, calculate a determinant by expansion, perform Gaussian elimination without and with pivoting. The students are familiar with the techniques and programming them enhances the comprehension of the mathematics. Being familiar with the techniques helps to understand how one structures an algorithm.

Since the course is very new, the second semester projects are not defined yet. Their level will be made dependent on the level of competence of the class and the individual candidates. While a minimal level project will be defined that average students can work on following clear instructions, the better students will be encouraged to define individual and more challenging projects. As far as possible, the projects will be drawn from the aerodynamics or structures classes of the second year.

Pre-requisite Knowledge

The mathematics course uses some of the first year knowledge, but is mostly independent. The computing relies on a half-module computing course (3h/week/semester) that introduces the students to programming techniques in general and MATLAB syntax in particular. MATLAB is used in all years and students directly entering the second year would be encouraged to get familiar with the language. Experience with the course so far shows that the students have extremely varied levels of competence in programming. Difficulties for some students need to be addressed by the research student in the lab and by tutorial sessions with small groups of students and the lecturer.

How Are Students With Different Mathematical Backgrounds Supported?

So far no direct entry students without a background in MATLAB needed to be supported, although this could be achieved with individual tutorial help. Different levels of competence are addressed by small group teaching in tutorial sessions for students with particular difficulties and by offering customised levels of difficulty in the projects.

What Support Was Needed?

The MATLAB teaching in the first semester is critical to achieving a programming competence that allows the tackling of interesting and applied projects. The level of achievement in the first year MATLAB course needs to be critically assessed and the level of the second level course adjusted accordingly. As it turns out, students achieved very different levels of competence which in turn required more individual attention in the lab session than was anticipated and more than the single research student could provide.

The Barriers

With the advent of powerful black box packages for structural analysis or CFD, students seem to tend to have an attitude that all the maths they need will be provided by these packages. Students clearly see the benefits of using the computer and IT skills have improved over the past years, but they seem less inclined to do programming. The weaker students who are struggling with the basics of linear algebra see their difficulties compounded by having to program it.

The Enablers

At this young stage of the course it is rather difficult to evaluate what is helping the students to succeed. Given the wide range of competence, key items are individual or small group tutorials to address specific difficulties. Small group tutorials “off the computer” are needed in addition to “online” help of a demonstrator for problems in the computer labs.

Evidence of Success

Success of the concept will be assessed partly from the student feedback and partly from the results in the math exams and the computing projects. Student feedback is sought verbally in pastoral tutorial sessions and from a standard university questionnaire at the end of the course.

For the next academic year a peer evaluation system is planned.

How Can Other Academics Reproduce This?

Basic proficiency in a mathematical programming language needs to be taught prior to using it in an applied context, but any package e.g. Mathcad should be suitable for the design of a similar course. Staffing with demonstrators needs to be sufficient such that students can maintain motivation rather than being frustrated by programming issues.

The level of the programming exercises needs to be carefully graded. Also students will achieve very different levels of proficiency and the setting of projects needs to be flexible to accommodate the varying abilities.

Quality Assurance

All QUB courses undergo a module review procedure at the end of the semester. Pathways and subjects are reviewed every few years. Students are called in for personal tutorials which assure the pastoral care and identify problems with a course early during the semester.

Other Recommendations

A “multi-disciplinary” course such as this will always require more attention in its development and maintenance compared to a pure maths or pure computing course. It requires enthusiasm from the lecturer, as well as good support from colleagues in providing examples, integrating other subjects or supervising projects.

Specialist Software

Use of Mathcad to Assist in the Teaching of Second Year Engineering Mathematics

Robert Beale ▪ Department of Civil Engineering and Construction Management ▪ Oxford Brookes University

Abstract

Mathcad is used in all years of the engineering mathematics course to enable students of civil engineering to investigate real engineering problems which have no analytical solution but which illustrate important mathematical concepts. In the second year engineering mathematics course Mathcad is used to assist in the teaching of numerical solutions of second order boundary value differential equations. Comparisons are made between classical analytical solutions and the numerical solutions.

Level of Material: Second Year

The Execution

In order to motivate the students to study engineering mathematics, problems drawn from other subjects studied in the degree are presented which require analytical and numerical solutions. An assignment is given to students early in the course which poses a problem drawn from structural engineering. It comprises a variable coefficient second order boundary value problem relating to beam-columns. The problem requires students to derive a set of finite difference equations for a limited number of internal nodes and solve the linear system of equations produced using Mathcad. Once this simple exercise has been undertaken the students have to extend their calculations to larger numbers of nodes and compare the results with approximate analytical solutions of constant coefficient differential equations which bound the true solution. The final part of the assignment which is normally only undertaken by students seeking high grades consists of then writing a Mathcad program using a ‘shooting algorithm’ for the same problem.

The exercise has the objective of making students realise that mathematics has a role in engineering, enabling practical problems to be solved. It revises the theory of second order differential equations which is needed at the end of the course when partial differential equations are encountered. It also extends their knowledge of Mathcad and its applications and consolidates finite difference theory.

The requisite mathematical knowledge, such as numerical methods for the analysis of differential equations, is taught by a series of lectures in parallel with the problem being given to the student. Additional Mathcad techniques required for the assignment such as handling large arrays are taught in problem classes.

Pre-requisite Knowledge

In the first year of the Engineering Mathematics course the students are introduced to Mathcad by an exercise which instructs them in some of the facilities of Mathcad such as equation solving, function plotting, etc. Students who enter the course directly into the second year frequently have not encountered Mathcad. A special demonstration of the facilities is given to these students and to all other students requiring revision or who did not fully understand all the features used in the first year.

The only other knowledge required of students before the second year is the solution of constant coefficient second order differential equations. This is taught to all first year students and is a pre-requisite for all students entering this module. Direct entry students into the second year without this knowledge are required to trail the course and study the first year module where this is taught.

How Are Students With Different Mathematical Backgrounds Supported?

By the time the students encounter the numerical solution of second order differential equations they have studied at least one prior mathematical module. Students from non A-level backgrounds are required to study an additional module in basic calculus techniques. Weak A-level students are counselled to take this module. During these prerequisite modules all the mathematical techniques required for the assignment not taught in the final engineering mathematics module are covered. Tutorials, surgeries and problem classes are used to develop necessary skills.

What Support Was Needed?

Courses in Mathcad are available from the suppliers Adept Scientific Ltd at regular intervals. However, Mathcad is reasonably easy to learn and the author trained himself very quickly. Technical advice on Mathcad is available from Adept and the suppliers are quick to respond, particularly to new users. Students using the software obtained tutorial support from the lecturer by e-mail. Additional support from the University Computer Centre would have reduced the academic demands but unfortunately there was no one in the centre with the skills to help. The University's response to the lack of support is that technical software can only be supported by the departments using it. Their support is for general purpose software such as Microsoft Office.

The Engineering Mathematics Course is taught by a specialist engineering mathematician with additional tutorial assistance provided by postgraduate research assistants.

The Barriers

The barriers to using Mathcad to enable the teaching of concepts of mathematics are firstly that some students are mathematically weak and see the computer as a hurdle to further progress and secondly that some students like to compartmentalise knowledge and do not want to make the effort to see the inter-relationship between different disciplines.

The weak students are encouraged in tutorial classes by making them appreciate the advantages of knowing the answers to mathematical problems before attempting to solve them. This increases their motivation and reduces resistance.

By being a practising mathematician researching and consulting in engineering I am able to give practical examples of the needs of Industry. This knowledge and appropriate examples tends to motivate, particularly the weaker students. Interestingly enough, mathematically strong students often would rather solve difficult mathematical problems than solve simpler problems which require interrelating different subjects.

If considering taking on this approach academics teaching engineering and science students should be encouraged to seek out problems from the subject specialists and integrate with the subject teams.

The Enablers

The major enabler for the course was the availability of the lecturer to get students over the initial hurdle. Once that had been achieved then students were sufficiently motivated to succeed. In addition sufficient terminals supporting the software had to be available and an appropriate support structure put in place.

Evidence of Success

The use of Mathcad has had two positive results. Firstly, students undertaking projects and assignments in the final year often use Mathcad without being prompted, even in preference to Excel. Secondly, because to program the finite difference equations meant that students had to get a full understanding of the technique, the coursework and examination components had better results.

How Can Other Academics Reproduce This?

Resources must be devoted to ensure that enough copies of the software are available so that students can always log on to use the package. Note that although the work reported here has used Mathcad, any computer algebra package which can undertake numerical calculations can be used.

Time must be spent in developing the assignments so that the students can see that the topics are relevant to their subject discipline. The assignments must then have a relatively simple starting point so that the students can achieve an initial encouragement before embarking on the full task.

Time must be allowed for students to learn the elements of Mathcad before being given too complex a task. In the current course this is achieved by teaching Mathcad in the preceding year to the major assignment.

Quality Assurance

All courses in Civil Engineering at Oxford Brookes University undergo annual feedback when all components of teaching and assessment are reviewed. The reviews of the Mathematics courses have commented favourably on the Mathcad components as providing relevance.

Other Recommendations

My main recommendations about using computer algebra packages to assist in the teaching of mathematics to scientists and engineers are:

- Produce a set of graded exercises to ensure that the students are fully familiar with the package before starting the assignment.
- Make sure that the assignment is subject specific and integrates several different topics.
- Provide enough copies of the software.
- Get other members of the subject team involved with selecting the assignment so that it is seen by students to be part of their subject development.
- Ensure that sufficient technical support exists so that student problems are solved quickly.

References

The Use of Mathcad in Service Teaching for Civil Engineering Undergraduates, CTI Mathematics Workshop on Innovative Uses of Mathcad in Teaching, Birmingham, in *Maths&Stats CTI Newsletter*, 7(4), p36, Beale, R. G., (1996).

Use of Mathcad in Mathematics Teaching to Civil Engineers in Proc. *Second Annual Symposium in Technology for Mathematical Learning*, June 1998, IC, London, p7-14, Beale, R. G., (1998).

Multiple Approaches

Using Technology to Teach Mathematics to First Year Engineers

Neil Challis and Harry Gretton ▪ School of Science and Mathematics ▪ Sheffield Hallam University

Abstract

This case study reports on the approach at one institution to helping first year engineering students to acquire the mathematical skills they need. The approach involves a range of support mechanisms, and the concerted use of technology as well as paper and pencil methods. Changes in curriculum, pedagogy and indeed assessment style have all proven necessary.

Level of Material: First Year

The Execution

Mathematical topics are treated according to 'SONG' – a combination of Symbolic, Oral, Numerical and Graphical approaches – broader than the traditional mainly symbolic approach, and in the same vein as with the Calculus Reform movement in the USA. Students are encouraged to engage in *doing* mathematics, and to exploit a range of technology throughout – graphics calculator, spreadsheet, Derive, pencil, etc. The rich interplay of graphic, symbolic and numerical approaches is emphasised. Technology empowers students to check their own solutions. Sometimes, mathematical ideas are introduced via modelling of an engineering situation. The 'Oral' refers to communicating mathematical ideas, formulating engineering problems and communicating the solution appropriately. Some relevant modelling case study assessments are thus used – with students encouraged to take full ownership of the problem, with some marks being given for demonstrating various key skills. Examples appear in the references.

Pre-requisite Knowledge

There is a widely recognised problem with what can be assumed as pre-requisite knowledge and skills for engineering students newly arriving at university. The problem arises from a complicated set of circumstances including: changes in pre-university mathematics, the diversity of backgrounds of students entering university, the need for development of curriculum (and staff) to make university expectations realistic and the impact of technology on what is really required. We hope for a certain level of numeracy, including sensible use of a calculator, a certain fluency in algebraic techniques, and some previous acquaintance with the ideas of the calculus. Many students in the group do not have A Level Mathematics, so we are frequently disappointed. It is a major task of the module to get the students to revisit and enhance previously encountered topics as well as moving onto new topics. The diagnostic plays a role here. We do not use the word *remedial* because of the possible stigma – each student revisits topics according to need.

Given the diverse intake, the module must strike a balance between, on the one hand getting students to cover the ground as laid out in the module document and on the other allowing each person to start from a position which makes sense to them individually. The curriculum is challenged by technology anyway – for instance how much of the traditional range of paper and pencil integration techniques must now be covered on paper before we will allow our students to feel comfortable with widely available CAS systems? We hope that students will benefit from a fresh approach to topics with which they may not have been entirely comfortable previously and we believe that motivation and interest are important alongside pre-requisite knowledge.

How Are Students With Different Mathematical Backgrounds Supported?

The whole module is predicated on a diverse intake, but there are additional measures in place. There is a range of support materials on paper and on-line, covering both module material and surrounding or prior topics. The initial diagnostic can set an agenda which can be supported through standard tutorials, the drop-in Maths Help, extra targeted tutorials and additional credit-bearing modules. Each worksheet has a range of problems and activities, with the aim of stretching the more advanced students, while giving all something in which to engage.

What Support Was Needed?

Key staff participate in, contribute to and learn from international conferences and national events, to exchange ideas and practice and to widen their perspective. Continuing funding and time is needed for this. In fact time for reflection and to interact with the academic community is the main resource needed. Tutorial staff and engineering staff affected by changing student skills have participated in local developmental events, for example, giving the opportunity to explore the implications of the technology. IT support is required to load, maintain and update software.

The Barriers

Students have preconceptions of mathematics and a broader approach which requires them to engage fully can provide a challenge for some. Some say, "I don't like to use technology until I understand the basics", but upon investigation it is not clear what "the basics" are. Some say "technology is just pressing buttons", failing to understand the challenging message that technology can help focus on the mathematical idea (although some do want to be taught which buttons to push!). Many students do not recognise the contribution this approach to mathematics can make to key skills development and tend to compartmentalise mathematics anyway.

Some staff have displayed a certain resistance to change, although when they explore and reflect on the philosophy and approach, the reservations often focus in on an unease about assessment. There are various examples of this: the use of calculators in examinations and the possibilities for "hiding" facts in the memory (but should we be testing memory?); or the value or otherwise of radical ideas such as learning diaries, which may enhance key skills and encourage more thoughtful learning, but which from a narrow focus may be seen as a distraction. Some staff found the loss of control – with students knowing the technology better than them – a difficulty.

The Enablers

The most important enabler is *enthusiasm* amongst staff. This has been an interesting experience of innovation, with the ideas being spread from a small group and adopted and adapted progressively and variously amongst both mathematics and engineering staff as they have had time to gain experience and to reflect. Small successes help things along, such as a student finding a novel way to solve a real engineering problem using technology, or an engineer seeing a way of integrating the new approach into a laboratory. It is important to be willing to discuss the approach openly with students, and we believe this reflection is an important part of learning.

Evidence of Success

We can only report qualitatively: to report on any improvement or otherwise in exam performance is not helpful as the nature of the assessment is changed when students are empowered by technology. As with all approaches, some students fare better than others – and it is best with those who engage fully. These latter have produced some remarkable work. Evaluation through quality systems, questionnaire, learning diary, and a more concerted set of group interviews give the diversity of views which might be expected. We do not claim to have the answer, but we have embarked on a very interesting learning process.

How Can Other Academics Reproduce This?

The main point is to reflect on the approach and how it might make a contribution within each person's context. Some might feel able to go further than others. But our experience is that if you wait for all to agree before you try something innovative then you will never change. To learn from and improve an innovative approach you need to try things and gain experience. Computers have been with us (and our students) substantially and increasingly for 40 years, and now real power is widely available and even hand-held, then we must deal with it. It is possible to experiment incrementally to some extent, but there comes a point where, for instance, someone has to propose and decide that graphics calculators are allowed in examinations.

Quality Assurance

Module feedback forms covering all aspects of Teaching Learning and Assessment (TLA) give scores much in line with other modules.

Other Recommendations

- Share ideas and materials, and be honest about both successes and difficulties.
- Put key handouts on both paper and as electronic files, but have a variety of other materials.
- Don't imagine any one thing – diagnostic, computer based learning, virtual environments, technology, drill and kill …- will solve all problems. The one universal thing is that students need human contact to help them face up to their mathematical problems.
- Present a range of approaches both technological and traditional and take account of diverse students with varying learning styles. You can bring them all to water, but some prefer it flavoured!
- Come and talk to us if you are interested!

References

"Expressive and explicit CAS – is it enough?", *International Journal of Computer Algebra in Mathematics Education*, 9(2), p155, Challis, N., Gretton, H., (2002).

"Diagnosing mathematical needs and following them up", in Manfred Borovcnik, Hermann Kautschitsch (eds): *Technology in Mathematics Teaching*. Proceedings of the ICTMT 5 in Klagenfurt 2001, Schriftenreihe Didaktik der Mathematik v. 25. öbv&hpt, Vienna 2002, Challis, N., Gretton, H., Robinson, M., Wan ,S., (2001).

"Technology, key skills and the engineering mathematics curriculum", Proc. IMA conference on *Mathematical Education of Engineers*, (IMA), pp 145-150, Challis, N., and Gretton, H., (1997).

"What is doing mathematics now that technology is here?", in Yang, W-C et al (eds), *Proceedings of the Fifth Asian Technology Conference in Mathematics*, ATCM Inc, USA, Gretton, H., and Challis, N., (2000).

Multiple Approaches

Process Systems Engineering – A Course in Computing and Numerical Methods for Second Year Chemical Engineers

Jack Ponton and Bill Morton ▪ Chemical Engineering ▪ University of Edinburgh

Abstract

We describe a course aimed at providing chemical engineering students with an understanding of the fundamental classes of equations which occur in chemical engineering, the mathematical basis of their numerical solution methods and the basic methods of implementing these in a high level computing language. The course thus integrates elements of both conceptual and practical mathematics and computing.

Level of Material: Second Year

The Execution

All non-trivial problems in chemical engineering practice are nowadays solved using numerical methods on computers. In 1993, we realised that numerical mathematics as taught by our mathematics department was having a negative impact and so we devised this module, taught by chemical engineers, addressing typical chemical engineering problems and requiring the students to write computer programs to solve them.

The problems were chosen to cover the range of types of equation: algebraic, o.d.e. (p.d.e. are left for an 'advanced' option), linear and non-linear and also simple linear and non-linear optimisation problems. From the engineering standpoint they also cover our major application areas including separation processes, reactors, material balances, and data fitting.

The emphasis is very much on practicalities. How do numerical methods work? What can go wrong? How do I write a program for this method? How can I adapt someone else's program? Only what we see as the very minimum necessary amount of theory is developed and issues such as numerical stability are mainly addressed by experiment.

In the full time module we include the teaching of Fortran 90. It is seen as somewhat controversial by some to continue to teach a formal programming language to all engineering students, but we believe that the development of an algorithm, which the writing of any program requires, is an important exercise in rigorous thinking which contributes to students' problem solving skills.

By 1998 we had most of the teaching material for the course on a web server and we decided to use this as the basis for a distance learning module to be offered, along with our web based control course, by Strathclyde University as part of their distance learning MSc in process technology. The major change, other than a different set of hand-in exercises, was to remove the formal programming element and provide the students with a set of web-based modelling tools which would generate models to run in a Lotus or Excel compatible spreadsheet.

Pre-requisite Knowledge

A minimum skill in algebraic manipulation, i.e. "changing the subject of a formula" along with the understanding of a derivative as a rate of change, are really the only absolute prerequisites. For the full time course all students have completed a Scottish First year or A level and are, at least in theory, familiar with the ideas of sets of equations, differentiation, integration and the analytical solution of linear first order o.d.e.s, and this knowledge is assumed in some problems.

How Are Students With Different Mathematical Backgrounds Supported?

For the full time course students have a homogeneous background. For the distance learning course the only assumed mathematical skills are those noted above.

What Support Was Needed?

The course depended critically on the presence of two staff members whose interests and research work lie in the field of mathematical and computing methods, and who believed in the course philosophy of teaching these in a practical and "hands on" manner. A computer lab which could accommodate the whole class was extremely important. Robust software had to be identified and/or developed.

The Barriers

Students do arrive with the idea that numerical mathematics is "difficult" and their earlier exposure to some concepts of numerical methods in A level and/or first year maths courses appears to have reinforced this misconception.

Although most students (and all of the distance learning course participants) are not very good at the mechanics of algebraic manipulation, it can be quite hard to persuade them to avoid it and allow the computer to carry out numerical calculations instead.

Most students hate being made to write programs since these must be absolutely correct; they find it hard to accept that an almost-working program is valueless.

The Enablers

The full time course has had mixed fortunes since its inception nearly ten years ago, but now receives good feedback from students and has a near 100% pass rate. We believe that the key to its present success lies in the workshop sessions; we have always believed that numerical mathematics must be learned by solving problems for oneself, and experience has borne this out. The workshops benefit from a well equipped lab with fast and reliable computers, something that we lacked in the early days.

The organisation of the workshops has also evolved from the early days. Originally students were required to hand in programs and write-ups after the labs. It soon became clear that cheating and copying were becoming endemic. We now require nearly all problems to be done in the lab under the eye of lecturers and tutors. Collaboration, as opposed to copying, is encouraged, and we see evidence of students helping each other in a positive way and even of groups forming spontaneously to tackle a problem jointly.

Evidence of Success

Full time course: In subsequent years students are required to use the techniques they have been taught.

Distance learning module: Feedback from participants has been very positive. Although many students initially expressed concern about their ability to return to a mathematically oriented subject after several years out of university, we have in fact achieved a 100% pass rate for the four cohorts who have taken the module.

How Can Other Academics Reproduce This?

Although tailored to chemical engineering, the underlying idea, that numerate engineering is ultimately about solving equations on computers, can be extended to other disciplines. Much of the background material, and certainly the web-based tools for model construction, are quite generic, and we have either placed these in the public domain or are prepared to licence them without charge.

The key to adopting our approach however, is the presence of an enthusiastic product champion.

Quality Assurance

The full time course is subject to internal and external QA procedures though staff-student liaison committees, student questionnaires, annual course review committee, etc. The degree programmes of which it forms part were given a "commendable" rating (the highest generally awarded) in the 2001 QAA review and our other, but rather more extensive, web based course in process control, on the experience of which we developed the present distance learning module, was awarded the exceptional "exemplary" rating.

The formal exam for the distance learning module is scrutinised by an external examiner. This year he commented that he thought the questions "..were rather more challenging than many others in the course"; despite this we had a 100% pass rate.

Other Recommendations

We add the following suggestions for anyone developing material for this type of module:

- Choose a set of engineering examples to illustrate the mathematical points.
- Make sure that the engineering content of these is very well known to the students; if necessary take time to describe this.
- If possible, extend one engineering example to cover more than one mathematical issue.
- Minimise the amount of theory described.
- For more complex examples, provide sample programs which students can modify.
- Make sure that the mechanics of computer access, printing etc., are as transparent as possible.

References

A public domain version of the teaching material (without hand-in exercises) is available linked from: http://ecosse.org/courses/

Some of the tools are linked from: http://ecosse.org/general/

Teaching Maths in Context

Signal/Digital Processing

MATLAB-Based Minimal-Mathematics Introduction to Engineering Topics

Ifiok Otung ▪ School of Electronics ▪ University of Glamorgan

Abstract

The problem of declining mathematical skills and appetite amongst university entrants is well known. In order to soften the impact that this makes on student recruitment and retention in the School of Electronics at the University of Glamorgan, it became necessary to explore a 'minimal-math' or 'engineering-first' teaching approach. MATLAB-based graphical user interfaces, simulations and animations are employed to give students an unclouded insight into the engineering concept and the underlying physical considerations, and a clear appreciation of the interplay of the parameters involved. This type of first encounter helps to stimulate the students' interest in the subject, erects crucial knowledge pegs, and lays a solid foundation to support a more mathematically rigorous approach during later encounters with the topic when any deficiencies in math skills will have been remedied.

Level of Material: First Year and Second Year

The Execution

This case study describes the teaching of multimedia communications, a module taken at Glamorgan by a very mixed cohort of honours and non-honours second-year students from various schemes, including Music Technology. The mathematical competence of the class is therefore varied, but generally significantly deficient in key areas. We attempt to deal with the situation by putting engineering first, aiming to first endear the students to the engineering concepts involved and their applications. In this way, maths is not inadvertently employed as a gatekeeper to the module, and an inappropriate level of maths content is avoided, which would leave especially this cohort of students feeling like they are being served a main meal of maths with some engineering for dessert. Once the students have an unclouded insight into the engineering concept and the underlying physical considerations, then some of the mathematical techniques required to precisely encapsulate the interplay of parameters are discussed. The approach generally avoids a lot of maths content, and aims to prepare the students to cope with the intellectual challenges facing the majority of practising engineers in the 21st century where ubiquitous software packages provide reliable solutions of common mathematical problems. To illustrate, consider the execution of the approach in teaching two common topics, namely signal sampling and frequency domain concepts.

To introduce signal sampling, we do not simply quote the sampling theorem and show its application. This would be tantamount to arming the students with a 'magic' formula that they can apply superficially to solve a standard engineering problem. Neither do we begin with a rigorous mathematical proof, involving a range of mathematical techniques, namely integration, Fourier transform, Parseval's theorem, impulse function and its characteristics, etc. This maths-first approach would be inappropriate to the cohort and many would disengage before reaching an engineering application of the mathematical discourse. Rather, we start the topic by discussing the engineering benefits of transmitting only the samples of a signal, and stir the students' interest and curiosity by asserting that the original signal can be perfectly reconstructed from the sequence of samples, provided we follow a certain rule. An extensive MATLAB-based animation lasting about 20 minutes is then employed to help the students to discover this rule, the type of device used for reconstruction and the natural penalty for flouting this rule. A foundation thus laid, the student can more confidently learn about anti-alias filter design, aperture distortion due to flat-top sampling and the practical issues involved in sampling. The maths involved in this subsequent treatment is mostly arithmetic and algebraic manipulations, and these are taught through worked examples.

To introduce the students to the frequency domain characterisation of signals, we do not begin with a mathematical discussion of the Fourier series, the Fourier transform and their properties. Rather, using a MATLAB-based graphical user interface, we enable the students to investigate a number of important topics including periodic signals and the sinusoid; Fourier synthesis and signal bandwidth; and time-frequency domain relationships. The software allows the student to pose and receive answers to a range of what-if questions, and to explore important engineering concepts. These include, among others, the effects of certain time-domain operations such as time reversal, signal delay and pulse shaping on signal spectrum. We impart this crucial appreciation of some of the key concepts that influence communication system design without the burden of abstruse maths. Emphasis is placed on the student acquiring a clear understanding of these concepts, while actual computation of Fourier coefficients is done using the MATLAB *fft* function.

Pre-requisite Knowledge

Because of the diverse nature of the cohort taking the module, little mathematical background is assumed beyond GCSE Maths. Most of the mathematical ability required is developed just-in-time for each topic. Arithmetic and algebraic manipulations, sine function, logarithmic measures, etc are taught at various stages and reinforced using worked engineering examples.

How Are Students With Different Mathematical Backgrounds Supported?

The maths required is taught at the point of need in an engineering context. Those students needing further help are identified during tutorial exercises and referred to the Education Drop-in centre. This is a central university resource located in the library and staffed four days a week for student consultation on maths problems. The centre also provides numerous free leaflets on a very wide range of maths topics that the students can pick up for home study.

What Support Was Needed?

In-house training is provided at the beginning of each academic year for research students involved in the delivery of laboratory and tutorial classes. The author has been developing programs in MATLAB since 1992, but MATLAB is very easy to learn especially for anyone with a computer programming background. Furthermore, MathWorks runs regular training courses in MATLAB and also provides technical support to registered users. Students using the software developed for the module need only basic computer literacy to manipulate the user-friendly graphical user interface.

The Barriers

An extensive use of the approach described in this case study is currently hampered by a lack of easy-to-use custom off-the-shelf software products designed to explore and simulate the engineering problem or concept of interest at a sufficient depth. In a classroom environment, nothing else beats a good computer simulation in engagingly demonstrating an engineering problem and the interplay of relevant parameters. However, developing the required software can be very time consuming and it is well worth searching the Internet for free web-based animations that may be relevant to the topic, although these will in general have some shortcomings.

The Enablers

There were sufficient terminals running MATLAB software that students could access in their spare time. Students were given a series of exercises to try out using the software to pose a range of what-if questions and record their observations. For example, how is the amplitude spectrum affected by time reversal? Time delay? Pulse shaping? Duty cycle? Etc. How is the phase spectrum affected? What signal is reconstructed from a sequence of samples of a given band-limited signal as the sampling frequency is changed? When does distortion set in and in what form? Etc.

Evidence of Success

Formal written feedback from a short course in digital telecommunications network for practising where the approach has been employed was very positive. Most felt that the approach made the subjects enjoyable and easy to follow. A similar evaluation will be undertaken at the end of the first year of adopting the approach for undergraduate teaching.

How Can Other Academics Reproduce This?

This engineering-first, maths-second approach is applicable to any introductory engineering topic especially if it is traditionally wrapped in maths. First, the discussion emphasises engineering considerations throughout and presents with computer simulations an intuitively satisfying and insightful treatment. Then and only then is maths brought in for a precise quantitative statement of the important parameters and applicable physical laws.

Quality Assurance

All modules offered in the School of Electronics at the University of Glamorgan are monitored and reviewed annually. Additionally, all assessment components are subject to both internal and external moderations.

References

"Reassessing the mathematics content of engineering education", Engineering Science and Education Journal, 10 (4), pp130-138, Otung, I. E., (2001).

"Computer-integrated lecturing in Telecommunications training", Proc. International Conference on Engineering Education, August 2002, Manchester UK, Otung, I. E., (2002).

Communication Engineering Principles, ISBN 0333775228, Palgrave Macmillan, Basingstoke, Otung, I. E., (2001).

Signal/Digital Processing

Analysing Random Processes using MATLAB at the Master's Level

John Soraghan ▪ Department of Electronic and Electrical Engineering ▪ University of Strathclyde

Abstract

MATLAB is the chosen simulation environment that is used throughout the Department of Electronic and Electrical Engineering. MATLAB is used by the students at several levels. It is used in earlier years as an 'Engineering' calculator that is useful for scientific calculations and visualisation particularly for complex analysis. As the course develops MATLAB becomes invaluable for investigating the time-frequency characterisation of signals and systems. MATLAB also gives the students an environment that allows them to write programming code in a 'C' like format. Finally MATLAB facilitates greater contextual teaching and problem based learning, which has become increasingly important in current Electronic and Electrical Engineering.

Level of Material: First Year (MSc) and Fourth Year (MEng) (Scotland)

The Execution

The case study involves the use of MATLAB in an attempt to give a better understanding of the fundamental correlation tools that are used to analyse random processes. In order for the students to be able to appreciate the MATLAB code associated with this study they need to have successfully carried out some tutorial questions related to correlation and random processes. Most mainstream engineering students find the mathematics associated with describing and analysing random processes particularly challenging. In our electronic engineering approach we establish correlation through the link with convolution and filtering (the matched filter). These are concepts that the students meet earlier in their courses and for which they have an implicit understanding. While the concept of a stochastic process is accepted by the students it is apparently much more difficult for them to grasp the mathematical tools that are available for analysing random processes. Having covered correlation and its mathematical representation the students are asked to learn how the autocorrelation, cross-correlation and spectral characteristics of a discrete signal are formed. This then forms the basis of the Wiener Khinchine theorem for estimating the spectral characteristics of a stochastic random process. It also allows the mathematical tools for identifying (modelling) stochastic processes to be developed. The basic MATLAB code that is provided to the students in this case study covers both these topics. The students are provided with the case study material early on in the courses and are encouraged to 'play around' with the MATLAB m-files using 'what if' scenarios. In their report they are requested to describe the underlying theory associated with the analysis of random processes and also to describe the MATLAB code and their changes to the code together with a discussion on their experimental observations.

Pre-requisite Knowledge

It is important that the students have a degree of ability with MATLAB and the use of m-files. Even for the weakest of students this appears not to be too problematic. The students need to have successfully covered linear systems theory to at least level 3 (third year Engineering in Scottish Universities). It is assumed that the concept of convolution and filtering is understood both at a mathematical level and contextual level. It is further assumed that the students have grasped the time – frequency dualities associated with signals and systems analysis. The fourth year Engineering Analysis module is made compulsory for the MEng students and optional for the BEng students. It is assumed that the MSc students have graduated with a first degree in Electrical Engineering at a level with at least 2.2 and that they have successfully completed a linear systems course on their first degree.

How Are Students With Different Mathematical Backgrounds Supported?

There is a wide range of mathematical ability within the typical fourth year Engineering Analysis class. The course was designed for the MEng student and particular emphasis has been given to mathematical rigour. In general the MEng cohort of students can cope with the mathematics and it is more an appreciation of the Engineering context that the mathematics fits into that they have problems grasping. The BEng students who opt to take the Engineering Analysis often struggle with some of the mathematical concepts. The use of appropriately phased tutorial questions is essential in order to address the ranges of ability and needs across the relatively large classes. It is also extremely important that properly prepared teaching assistants are employed in the tutorial sessions to address the students' problems. It is often the case that the student can gain more from the answer given by the teaching assistant than that by the lecturer! Finally, the use of email allows the students to get rapid on-line tutorial/classwork.

The students on the Communication, Control and Digital Signal Processing (CCDSP) MSc exhibit a very wide range of mathematical ability. It is important that one-to-one links are made with the relatively small class sizes (~15-20) in order to improve the weaker students' ability. This is again achieved through appropriately phased tutorials and through the email hot line.

What Support Was Needed?

As no formal laboratory sessions accompanied these modules it was important to have a sufficient number of PCs available with the MATLAB environment that will allow the students to experiment with the code at a time and with a pace that suits them. Furthermore it is important that these labs are accessible by the students outside lecture hours. The use of teaching assistants that have the necessary background was fundamental to achieving the learning objectives of the modules. In the case of this study the students needed to have a solid understanding of digital signal processing theory and the MATLAB environment.

The Barriers

The main barrier was the time management skills of the students. It was expected that students would work on material over a period of six weeks. As a result it was anticipated that students would endeavour to accomplish the set objectives by spreading the workload. Due to competing demands from other courses it was found that students often left the work towards the end of the time period.

The Enablers

There is a departmental wide MATLAB site licence in place for using the software for undergraduate teaching. The PC labs are also plentiful and are open until 9pm.

The teaching assistants are recruited from the Institute of Communications and Signal Processing where most researchers use MATLAB in their research. There is a strong Signal Processing Group within this institute, which facilitates good contextual based teaching. The quality of the MEng intake and those taking the CCDSP MSc is very high.

Evidence of Success

It is apparent that the students gain doubly through this case study. Firstly it gives them some hands-on programming which they can import and exploit into their fourth year project work or MSc project work. Secondly and most important it allows the lecturer to bring mathematical concepts associated with random signal analysis into a graphical user interactive simulation arena.

How Can Other Academics Reproduce This?

Appropriate resources must be available for the concept of this approach to project based learning to be successful. Students must be comfortable with MATLAB but not experts! Good quality and motivating research assistants are vital for success. Finally the students that the material described in this study is aimed at are highly qualified.

Quality Assurance

There is continual feedback from the students during the tutorials and their tutorial submissions. Important feedback is obtained from the one-to-one email correspondence that takes place between the teaching staff and the students. There is also a formal QA form that is completed at the end of each course within the Department. This contains useful comments regarding the course material and ways in which it could be enhanced. It is apparent that the students have responded very favourably to MATLAB as an aid to mathematical understanding.

Other Recommendations

- Don't over stretch the students with a piece of work that is going to extend over half a semester in one module.
- Ensure from the outset that ALL students know exactly what is expected from the course material and what they can expect from the teaching staff.
- Employ an early feedback mechanism that allows tracking of the students' efforts or lack of effort on the material.
- Make sure ALL teaching staff are very comfortable with the subject material.

References

DSP First: A Multimedia Approach, ISBN: 0-13-243171-8, McClellan, J., Schafer, R., and Yoder, M., (1999).

Discrete-Time Signal Processing, ISBN: 0-13-754920-2, Oppenheim, A., Schafer, R., and Buck, J., (1999).

Discrete Signals and Systems Analysis, Lecture Notes, University of Strathclyde, Oct 2001, Soraghan, J., (2001).

Physical Chemistry and Materials

Teaching Mathematics to First Year Undergraduate Chemists in the Context of their Discipline

Paul Yates ▪ School of Chemistry and Physics ▪ Keele University

Abstract

New entrants to chemistry degree programmes are given a 24 hour course in mathematics if they do not have an A level qualification in the subject. This concentrates only on the skills necessary to successfully complete the first year physical chemistry course; these include simple statistics, functions, partial differentiation and integration. The course is taught using chemically relevant examples, in an order related to the chemistry course rather than traditional mathematics courses.

Level of Material: First Year

The Execution

Students arriving at university to study chemistry without an A level mathematics qualification are frequently surprised to learn that they will be required to take a course in mathematics for chemists. As they will have given up the subject two years earlier they are unlikely to be highly motivated on such a course, so retaining their interest and commitment is paramount. In order to do this, the course is based on the chemical topics to be taught which dictate both the order in which the mathematical material is covered and the examples which are used to illustrate the various mathematical topics. This has the advantages of both ensuring that the material being taught is highly relevant and of introducing students to material in the physical chemistry course which will be built upon later. This is a useful additional benefit that should aid learning at a later point in the course.

A potential disadvantage is that a lot of material is covered at an early stage of the undergraduate career and some of this is quite advanced. For example, partial differentiation has to be taught to students with only GCSE mathematics in their first semester. However, the requirements of the physical chemistry are such that the most important aspect is for students to be able to recognise appropriate symbols when they are met, rather than being necessarily able to perform certain skills. Thus exposure to a wide range of techniques can be considered appropriate even at this early stage.

Pre-requisite Knowledge

University matriculation requirements ensure that students will have at least GCSE mathematics at grade C. No specific prior knowledge is assumed, other than the ability to perform simple arithmetic and the simple use of a calculator. Instruction in more advanced calculator use is provided as appropriate during the course and at an individual level during problem classes.

Timetabling considerations result in the first contact session of the course being a problem class. In this session students work through context based problems that require only elementary mathematical skills. This is designed to get them to think about chemistry in numerical terms.

Basic computer use is assumed in order to run the computer assisted learning tutorials. Such introductory courses are provided centrally by the university at the start of each academic year. Students are made aware of these and are strongly advised to attend if appropriate. The presence of a member of lecturing staff during the formally timetabled computer tutorials ensures that minor difficulties in computer use are rapidly resolved.

How Are Students With Different Mathematical Backgrounds Supported?

The course is typically taken by students whose mathematical qualifications range from GCSE grade C to A level grade D. The latter group of students typically find the course much easier as it is largely reinforcing material they have previously studied. The presence of more than one member of staff in the problem sessions, and the fact that these staff walk around the classroom and are proactive in helping students, ensures that support is targeted at the weaker students.

Those students who have A level mathematics at grades A-C would not generally take this course, and would not generally receive any specific mathematical support. The one topic taught in this course which they would not have covered is partial differentiation, so this is actually covered at the appropriate point in the physical chemistry course. This also has the advantage of revising this conceptually difficult topic for the other students immediately prior to its use. The provision of weekly drop in sessions will also provide support for students who do have A level mathematics.

What Support Was Needed?

The course is taught by several members of the physical chemistry lecturing staff, depending on their availability and teaching loads in a given year. Problem sheets are prepared by the author and used by all the staff teaching on the course. Support for the problem classes has been provided by a postdoctoral fellow in physical chemistry, but in some years there have been delays in agreeing funding for such support. Also, when this member of staff left it was difficult to find a suitable replacement with a sufficiently high level of mathematical competence.

Development of the computer assisted learning software took place within the Chemistry Courseware Consortium, so the only input required was academic, involving the production of suitable scripts. All programming and implementation was performed by specialist staff at the CTI Centre for Chemistry at the University of Liverpool. Mounting of software on the departmental PCs was performed by the author. Maintaining this software was a time consuming task, although the department has now assigned a member of technical staff this responsibility.

Producing the textbook required the usual authoring process, but was hindered by frequent changes of personnel at the various publishers who were involved due to mergers and take-overs.

The Barriers

Many chemistry students are mathematically weak. However, a more significant barrier is that they may not appreciate the need to acquire certain mathematical skills. The approach taken here is designed to overcome this by demonstrating the relevance of mathematics to the physical chemistry course being studied.

The Enablers

The most important aspect of this course is the availability of sufficient staff to deal with student problems on an individual basis. Although this is largely done during the problem classes, provision of computer tutorials and drop in sessions also provide valuable opportunities for staff-student interaction.

Evidence of Success

For the motivational reasons noted, this would not be expected to be a particularly popular course. However, confidential student feedback has been consistently positive over a number of years. Failure rates are relatively low and most students go on to complete the first year physical chemistry course successfully. External subject reviews of the course have provided very favourable feedback.

How Can Other Academics Reproduce This?

The critical requirement in order to reproduce this form of course for other disciplines is a sufficient supply of discipline related problems. These can generally be generated by taking typical problems within a discipline and providing sufficient further information so that only mathematical skills and knowledge are being tested.

The provision of related software and textbooks is more difficult and will depend on individual circumstances, but may be possible once the problems are available.

Quality Assurance

The course has been subject to standard departmental quality assurance controls. These involve annual monitoring of student feedback on the overall module and a report by the module leader.

Other Recommendations

I would recommend using discipline specific mathematical problems to both illustrate and structure a support course in mathematics. In addition

- Provide sufficient staff support.
- Provide a variety of opportunities for staff-student interaction.
- Ensure that technical support is available.

References

Chemical Calculations, Blackie, Yates, P. C., (1997).

Chemistry Courseware Consortium, 1998. The Chemistry Tutor, Houghton-Mifflin. *Learning Hierarchies in the Physical Sciences*, http://www.keele.ac.uk/depts/ch/ltsn/home.html

Mathematics in Context. Education in Chemistry pp 78-80, Yates, P. C., (2002).

Physical Chemistry and Materials

Teaching Mathematics to Chemistry Students at the University of Sheffield

Michelle Webb ▪ Department of Chemistry ▪ University of Sheffield

Abstract

The department of chemistry offers over two semesters, Mathematics for Chemists 1 and 2, which provide students with the understanding and use of mathematical techniques for various chemistry degrees. This case study reviews these courses and illustrates their value in terms of providing the students with a positive foundation for future study.

Level of Material: First Year

The Execution

In the Department of Chemistry, the courses Mathematics for Chemists 1 and 2 are offered to first year students. The aim is to provide undergraduates with the understanding and use of mathematical techniques needed for various chemistry honours degrees.

Taught over two semesters by different tutors, each course runs for 12 weeks with two lectures a week and one tutorial. To monitor their progress the students are assessed every six weeks. Each lecture is designed to spend thirty minutes on basic mathematical principles and the remainder is taken up with questions. Questions are set each week to accompany the lectures and they are discussed in a tutorial the following week.

At the beginning the students are given a preliminary assessment. The topics range from basic arithmetic through to integration and differentiation. The tutor uses the results as an indication of the strengths and weakness within the group; the results are not part of the overall assessment. Students who do very well in the test have two options: they can stay and mentor the other students in the class or they can find another 10-credit module and take up the course in the second semester. Many stay on and take up the role of mentor.

During the first semester Mathematics for Chemists 1 starts with basic arithmetic and covers topics from the GCSE syllabus up to and including A-level. For the first six weeks the subjects include algebra, fractions and surds, linear equations, quadratics and simultaneous equations. The students then sit a test.

The next set of topics includes exponentials, logarithms, statistics, geometry, trigonometry and differentiation. At this stage, the students sit another test.

The second semester topics (Mathematics for Chemistry 2) are more advanced (integration, matrices, vectors etc.) and the mathematics is more chemistry based.

Where possible, every effort is made to incorporate mathematical problems in a chemistry context. Following the section on basic algebra, the students are presented with the re-arrangement of equations and fractions using chemistry equations. In doing quadratics and solving simultaneous equations, chemistry examples are also used. Figure 1 illustrates an example used in a trigonometry tutorial, which takes the student from the basic mathematics to calculating the hydrogen-to-hydrogen distance in the water molecule and calculating the 0-0 bond length in the ozone molecule.

The objective is to introduce the basic topics and establish a contextual link to the chemistry. The student can then see the role mathematics plays and broaden their understanding. Each course is designed to help the students as many are lacking in confidence and ongoing support is required. It is therefore important not to make the mathematics complicated but to create a foundation for future study and to let them see that it can be fun.

Pre-requisite Knowledge

The course caters for a variety of backgrounds. Essentially any student doing a chemistry course who does not have mathematics will take one or the other of the semesters depending on the level of maths. Those who do not have A-level mathematics will take both courses (Mathematics for Chemists 1 and 2) while students who have AS-level mathematics will take the course Mathematics for Chemists 2 in the second semester.

What Support Was Needed?

- Mentoring for the first semester is an important role in the course for guiding the weaker students. Mentors are set up unofficially in the class.
- Photocopies of each of the tests from the preliminary assessment are given to the student's personal tutor. This helps them to appreciate the level at which support is needed by the student during the semester.
- The students complete a questionnaire relating to the course. The responses are put up on a large notice board in the department. It is the responsibility of the staff to respond to these comments and the students get to see their written replies. Staff do respond to the students' concerns offering support and this can lead to changes in the course structure and teaching method.

The Barriers

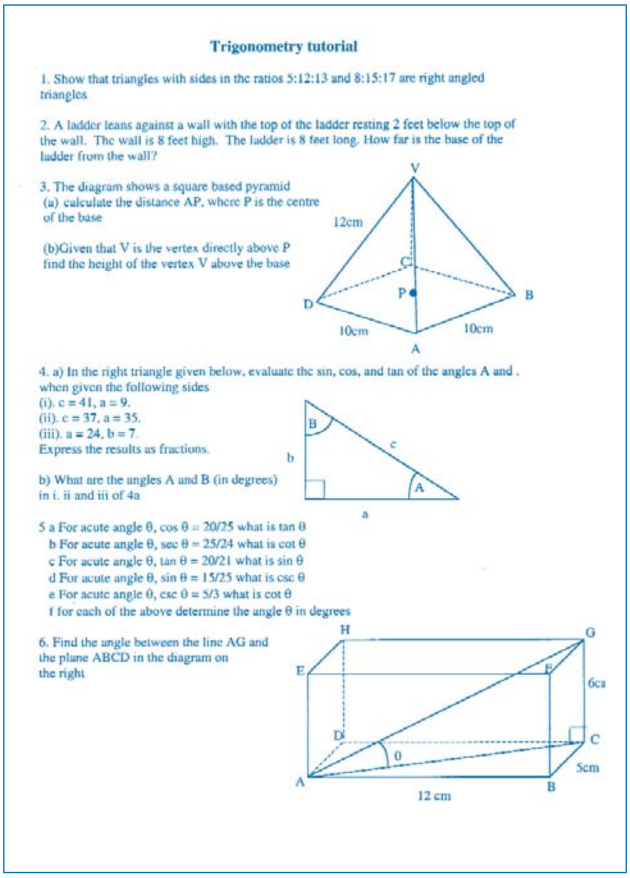
The main barriers are time and resources. The setting up and co-ordination of the courses require ongoing attention. There have to be regular updates in the content and the support as the tutors face continuing changes in the level of mathematical skills.

Evidence of Success

The continual assessment of the students indicates improvement during the year. Feedback from the students is extremely positive: “This was taught very well and I understood everything as I went along. Being examined half way also meant I understood more rather than just memorised content. Tutorials gave practice in rules and helped with revision”. Another stated “The course has helped my maths in chemistry”.

How Can Other Academics Reproduce This?

- Teaching resources are required to set up the programme.
- The two-semester courses Mathematics for Chemists 1 and 2 are structured to provide for different levels of mathematical skills amongst the students.
- The tutors require an understanding of the current A-level and GCSE curricula.
- The preliminary assessment is important in setting up the content of the early lectures.
- The courses must accommodate for the wide level of ability, those acting as mentors are set difficult questions and the weaker students are advised “not to worry if they cannot do them”.



Trigonometry tutorial

1. Show that triangles with sides in the ratios 5:12:13 and 8:15:17 are right angled triangles

2. A ladder leans against a wall with the top of the ladder resting 2 feet below the top of the wall. The wall is 8 feet high. The ladder is 8 feet long. How far is the base of the ladder from the wall?

3. The diagram shows a square based pyramid
(a) calculate the distance AP, where P is the centre of the base

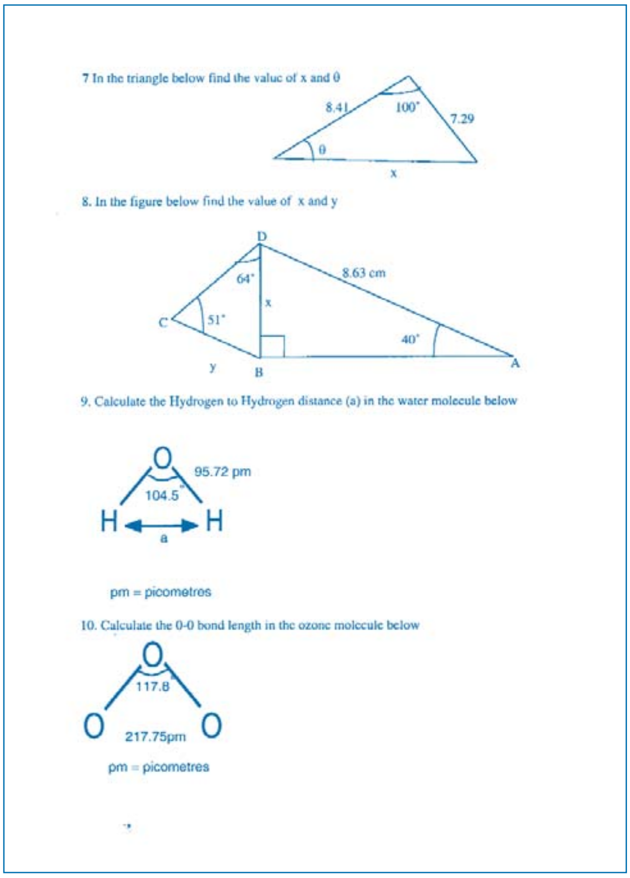
(b)Given that V is the vertex directly above P find the height of the vertex V above the base

4. a) In the right triangle given below, evaluate the sin, cos, and tan of the angles A and . when given the following sides
(i). c = 41, a = 9.
(ii). c = 37, a = 35.
(iii). a = 24, b = 7.
Express the results as fractions.

b) What are the angles A and B (in degrees) in i. ii and iii of 4a

5 a For acute angle θ, cos θ = 20/25 what is tan θ
b For acute angle θ, sec θ = 25/24 what is cot θ
c For acute angle θ, tan θ = 20/21 what is sin θ
d For acute angle θ, sin θ = 15/25 what is csc θ
e For acute angle θ, csc θ = 5/3 what is cot θ
f for each of the above determine the angle θ in degrees

6. Find the angle between the line AG and the plane ABCD in the diagram on the right



7 In the triangle below find the value of x and θ

8. In the figure below find the value of x and y

9. Calculate the Hydrogen to Hydrogen distance (a) in the water molecule below

pm = picometres

10. Calculate the 0-0 bond length in the ozone molecule below

pm = picometres

Figure 1: Trigonometry Tutorial Example

Physical Chemistry and Materials

Teaching of Chemical Thermodynamics using Available Data and an Innovative Approach

Arthur Burgess ▪ Department of Contemporary Sciences ▪ University of Abertay Dundee

Abstract

Analysis is made showing how Helmholtz and Gibbs energies conveniently interrelate enabling typical 2-D and 3-D curves to be drawn across a range of temperature for selected chemical equilibria. Opposing influences leading to a free energy minimum or an entropy maximum are given a physical explanation with the attainment of equilibrium and the choice of conditions made evident. Simplifying assumptions are emphasised and the examples show how the data are manipulated, limits evaluated and trends in equilibrium summarised by EXCEL charts.

Level of Material: Second Year

The Execution

Chemical thermodynamics seeks to make some sense out of the nature of chemical reactivity and in particular the circumstances that bring change to an individual reaction and govern the condition known as chemical equilibrium. Undoubtedly the progression to a rounded view of equilibrium thermodynamics comes with a fuller appreciation of the mathematics applied. This develops as students seek out relevant data to analyse using appropriate relationships and techniques for evaluating limits, testing maxima and minima and processing with EXCEL or similar.

The approach evolved as students showed a preference for accessing their own data in lecture examples and tutorial problem solving. Lab programmes were adjusted to suit convenient data acquisition using familiar reaction processes such as volatile liquid vaporisation and more time was released for data manipulation and mathematical processing. Generally practical work allows less time for the data analysis and often proceeds with linear plots to determine for instance enthalpies of vaporisation. With a different emphasis the same data can be treated more extensively to produce a family of dipping free energy curves portraying typical equilibrium minima over a range of temperatures. As well as being visually more attractive this analysis gives quantitative information on how equilibrium minima shift with temperature rather than qualitative indications associated with Le Chatelier type predictions based on enthalpy change alone.

Traditionally chemists concern themselves with the Gibbs energy function almost exclusively for describing reaction equilibria and assert that the Helmholtz energy function is less useful in chemistry because processes and reactions are more often carried out at constant pressure than at constant volume. This view however is unnecessarily restrictive and leads to a lack of use of Helmholtz energy to describe chemical systems such as vaporisation and thermal decompositions where a change of perspective from Gibbs energy is particularly informative and appropriate. The method builds on the familiarity with Gibbs energy, its relationships, approximations, conditions and transformations applied in dealing with reactivity of a chemical process and the establishment of dynamic equilibrium. A wealth of thermodynamic data is available and listed for convenient access in summary form particularly at 298 K fixed temperature and 1 bar standard pressure. Thus standard Gibbs energies of reaction are easily calculable at 298 K and at other temperatures with reasonable accuracy by further processing. However to apply these data modifications to depict how Gibbs energy changes during a constant T,p reaction we find that unless mixtures of gases are involved then we cannot illustrate free energy descent to a typical equilibrium minimum (a highly emphasized feature of equilibrium thermodynamics). This restriction may be overcome and so encompass a much wider range of reactions by transforming the data set from Gibbs to Helmholtz energies and considering the consequence to the free energy profile for reactions maintained at constant V,T. Such a shift allows quite an intriguing insight into the circumstances that bring about reactants changing to products then establishing equilibrium at a position intermediate to full conversion. Certainly this gives a powerful demonstration of the enabling mathematics to model such processes and is firmly based on evidence and experience gained rather than allowed to remain as concepts within theoretical discussion.

Pre-requisite Knowledge

By the end of the first year students have encountered the analytical methods required and have reasonable confidence with data analysis packages and of seeking IT/mathematical support. Nevertheless care is taken to reconsider the methods appropriate to the task in hand and this can be usefully combined with introductory discussions to the laboratory/tutorial groups undertaking particular reaction investigations.

How Are Students With Different Mathematical Backgrounds Supported?

Pre-course and remedial classes during the first year contribute together with availability of personal skill development in tutorials, surgeries and general accessibility to maths/IT staff.

What Support Was Needed?

Discussion with other staff helped give shape to the approach gradually adopted. It was decided early on that the data analysis and information portrayal should be conveniently managed by using EXCEL. Technical support with some of the teething problems was provided by IT staff and this extended the analysis to other reaction types as further aspects of study became apparent.

The Barriers, Enablers and Evidence of Success

There is some initial resistance to using Helmholtz energy, a thermodynamic function that is relatively unfamiliar and often taken at face value to be of secondary importance to Gibbs energy in contributing to an understanding of chemical reactivity. This is not helped by the sparse development of the Helmholtz energy function generally given by most undergraduate textbooks. Conversely though this helps bring freshness and an element of research to the application in the lab and problem solving. This becomes apparent when the information aspects obtained from the interrelationship of the two free energy functions can be readily accessed and convincingly portrayed.

How Can Other Academics Reproduce This?

A more detailed description of the methodology is available with examples of reaction types and interpretation of results from free energy profiles showing minima corresponding to dynamic equilibrium positions and related entropy terms producing maxima. This information can be found at the following URL **www.mathcentre.ac.uk/resources/casestudy/burgess_a_casestudy.asp**

Quality Assurance

The lab reports showed a freshness and individual commitment towards the assignments. Thermodynamic questions in examinations requiring data acquisition were more frequently attempted, more detail presented and though inappropriate data could be selected the method of approach to problem solving could be credited and a general improvement in answer standard noted.

Other Recommendations

Availability of an adequate data book in lectures, labs and tutorials on an individual basis was essential and opened up opportunities to show how data might be accessed from different tables and the same data directed to portraying different aspects and modified for other physical conditions.

In dealing with data manipulations with terms arising that show indeterminate values for the EXCEL charts it is a good opportunity to highlight methods of obtaining limiting values (a particular case is for terms similar to $x \ln x$ where x is at or near to zero).

Opportunities exist to consider a variety of reaction types and propose more investigative content for lab/tutorial assignments. On the other hand a fairly wide choice of liquid equilibrium vapour pressure data can be sourced in addition to sublimation for similar examples that have relatively few terms for testing in their defining equations.

Simplifying the defining conditions is recommended and in line with other physical approximations means for example real gases are taken in a range where ideal gas behaviour can be assumed. Again to keep the equation form as simple as possible a pressure limit that a vapour might reach is selected to be the same as the standard pressure of 1 bar.

References

Chemistry Data Book 2ed, John Murray, London, Stark, J.G. and Wallace, H.G., (1989).

Perry's *Chemical Engineers' Handbook* 7ed, Green, D.W. editor McGraw-Hill (1997).

CRC Handbooks of Chemistry and Physics 80ed, Lide, D.R. editor-in-chief (1999).

The A to G of Chemical Equilibrium: aspects depicted by Helmholtz energy using its relationship to Gibbs energy, *Journal of Chemical Eduction*, Burgess, A. – to be published.

Physical Chemistry and Materials

Mathematical Methods for Third Year Materials Scientists at Cambridge University

Interview with John Leake ▪ Department of Materials Science and Metallurgy ▪ University of Cambridge

Abstract

Mathematical Methods is a revision course for third year materials scientists. Started in 1997, there is no formal examination. It consists of six lectures, an examples class and a questions sheet, and provides revision of past topics, with examples relating to third year materials courses and a background for the fourth year. This case study reviews the course and its role in providing the student with a mathematical foundation in the context of materials science.

Level of Materials: Third Year

The Execution

The course Mathematical Methods is taught to third year students as part of a four-year degree in Materials Science and Metallurgy. The principal aim is revision; lectures concentrate on specific mathematical topics but they also draw on examples taken from materials science, where the mathematics is illustrated as a tool. There is no direct course assessment.

The course consists of a compact series of six lectures, which take place at the beginning of the academic year, over two and a half weeks. The lectures are based on the following programme: Introduction (2 lectures revision of several short, important topics), Matrix Algebra (1 lecture), First-Order Differential Equations (1 lecture), Diffusion (1 lecture) and finally Error Analysis (1 lecture). A 70-page handout containing in-depth materials covered in the lectures is given to each student.

Another handout is given to the supervisors, in which there is a questions sheet and a set of questions for the examples class which takes place a week or ten days later, with detailed solutions to all the questions. The examples class and the question sheet operate in parallel and cover topics presented in the lecture course. A designated supervisor will ask the students to attempt the question sheet before a certain date. Each supervisor has the responsibility of providing support and tuition for two or three groups, each containing two or three third year students. Any problems or questions will be dealt with during supervisory sessions.

Teaching in each of the topics involves shifting the context from pure mathematics to a more physical application. For example, the materials on approximations, based on series expansions covered in the introductory lectures, addresses the effect of an electric field on the movement of an ion in a solid. In another example, which looks at the use of the geometric series in materials science, the students investigate the calculation of several aspects of the molecular weight distribution of a polymer during stepwise polymerisation.

Throughout the course, the students are making use of equations they have seen in previous years. Figure 1 shows an example on crystal growth. In this instance, the students will have seen a similar equation in the lecture course on phase equilibria in the first year course Part IA Materials and Mineral Sciences. Such questions cover a broad spectrum of knowledge from first year mathematics through to topics in materials science, which are presented in context to illustrate the application of the mathematics.

The principal aim of the course is revision, but it is more than a review of past topics. It complements first and second year mathematics courses, runs in parallel to the third year and provides background knowledge for the fourth. The supervision and examples class provide ongoing support and a chance to resolve difficulties. Overall, the course reinforces the importance of mathematical competence when addressing materials science problems wherever they arise. By the end of the course, the students have a broader understanding and more confidence in dealing with the mathematics.

Pre-requisite Knowledge

The background of the students entering the course varies. Approximately 160 - 180 first year students take Materials and Mineral Sciences as part of a combination of subjects e.g. with Physics and Chemistry plus a course in Mathematics for physical scientists. 30 - 40 of these students then enrol in materials science in the second year, again as part of a combination of subjects. During this repositioning some undergraduates decide to specialise in materials science in their third and fourth years. Mathematical Methods is designed to enable these students, although they are from various backgrounds and levels of mathematical experience, to handle the mathematical content of the course. Except for a very small number of students transferring from Engineering to Materials Science at the start of the third year, all students who take the course will have taken the first year course in Mathematics for Natural Sciences, which is now compulsory. A few will have taken Mathematics as one of their second year subjects along with Materials Science and one other subject but knowledge of that course is not a pre-requisite.

What Support Was Needed?

The team of supervisors provides ongoing support for the students. Problems are addressed as the students work through the question sheet and the examples class questions. At the end of the term, the supervisors produce a report commenting on individual students' progress. These are placed online. Each student can access his/her own reports, as can their designated academic advisors (Tutor and Director of Studies).

Evidence of Success

There is no direct measurement of the student's achievement in the course. There is an indirect measurement in terms of the performance in other courses.

In third and fourth year courses the students are expected to handle appropriate mathematical calculations in the examination. Although it has not been formally measured there is a clear indication that competence levels have improved.

The ability to cope with mathematical components in lectures, practicals and projects across the courses during the third and fourth year has also shown signs of improvement. Supervisors have noticed that whilst the maths may be a challenge students are better able to cope. All told, there is enhanced performance and a willingness to handle the mathematical problems.

How Can Other Academics Reproduce This?

The first step is to identify the courses where the maths is essential, in this instance materials science. Key mathematical topics must also be identified in the respective undergraduate courses. Once identified these topics will be covered in the lectures, the question sheet and the examples class in a way that makes direct application to the subject apparent. It is important to illustrate to the student that the topics covered are directly applicable to the subject. For those institutions that do not have a supervisory system in place extra example classes would be required.

Quality Assurance

All courses have standard format questionnaires, filled in by the students, and these are circulated to the Teaching Committee. The committee also provides the department with reports from the Internal and External Examiners highlighting particular features of the examination. Comments on a particular maths topic, which students are finding a problem, are fed back for those co-ordinating the course to consider. In addition, there are consultative meetings at the end of the main terms with representatives from each of the undergraduate years. Frequent contact with the supervisors, including informal contact in the departmental Common Room, provides a valuable source of comments based on individual student progress.

1. Crystal Growth – a simple model

1.1 By considering the difference between the rates at which atoms jump from liquid to solid and from solid to liquid across an interface, show that the growth rate, U, of a crystalline solid into a liquid of the same composition is given approximately by

$$U = \lambda \nu_j \left(\frac{v\Delta G_v}{kT} \right) \exp\left(\frac{-Q}{kT} \right)$$

where λ is the atomic diameter, ν_j is the attempted 'jump frequency', $v\Delta G_v$ is the free energy decrease per atom on solidification, Q is the activation energy for diffusion in the liquid, and it can be assumed that $v\Delta G_v << kT$. [An equation very similar to this was considered in the lecture course on phase equilibria in Part IA Materials and Mineral Sciences.]

1.2 Recalling from Part IA Materials and Mineral Sciences that ΔG_v is a function of temperature, derive an expression for the temperature at which U is a maximum.

1.3 Typically, $Q \sim 1$ eV per atom and $\Delta H_f \sim 0.1$ eV per atom for solidification. Using your result in 1.2, show that the assumption that $v\Delta G_v << kT$ at the temperature where U is a maximum is valid for a solid with a melting temperature of 1000 K.

Figure 1: Crystal Growth Model

Index

LTSN Engineering

Tel: 01509 227170
Fax: 01509 227172
Email: enquiries@ltsneng.ac.uk

www.ltsneng.ac.uk

LTSN Maths, Stats & OR Network

Tel: 0121 414 7095
Fax: 0121 414 3389
Email: info@mathstore.ac.uk

http://ltsn.mathstore.ac.uk

LTSN Physical Sciences

Tel: 01482 465453
Fax: 01482 465418
Email: ltsn-psc@hull.ac.uk

www.physsci.ltsn.ac.uk

UK Centre for Materials Education

Tel: 0151 794 5364
Fax: 0151 794 4466
Email: ltsnmat@liv.ac.uk

www.materials.ac.uk

ISBN 07044 2374X

Also available in the series:
diagnostic testing for mathematics
maths support for students