Differentiation

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
k, constant	0
x	1
x^2	$2x$
x^3	$3x^2$
x^n, any constant n	nx^{n-1}
e^x	e^x
e^{kx}	ke^{kx}
$\ln kx = \log_e kx$	$\frac{1}{x}$

The sum-difference rule

$$\frac{d}{dx}(u(x) \pm v(x)) = \frac{du}{dx} \pm \frac{dv}{dx}$$

Constant multiples

$$\frac{d}{dx}(k \times f(x)) = k \times \frac{df}{dx}$$

for k constant

The product and quotient rules

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx} \qquad \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

The chain rule for differentiation

If $y = y(u)$ where $u = u(x)$ then $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

Integration

$f(x)$	$\int f(x)\,dx$
k, constant	$kx + c$
x	$\frac{x^2}{2} + c$
x^2	$\frac{x^3}{3} + c$
x^n, $(n \neq -1)$	$\frac{x^{n+1}}{n+1} + c$
$x^{-1} = \frac{1}{x}$	$\ln x + c$
e^x	$e^x + c$
e^{kx}	$\frac{e^{kx}}{k} + c$

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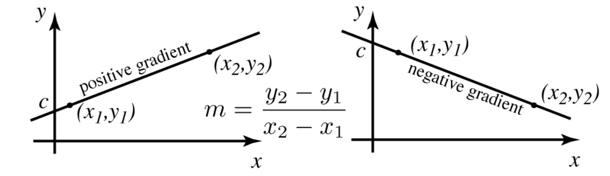
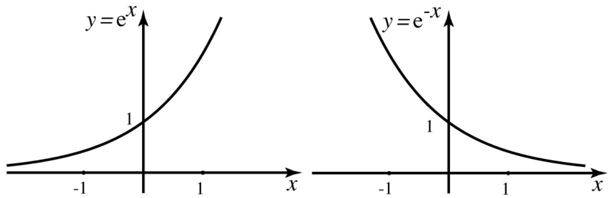
Graphs of common functions

Linear: $y = mx + c$, m=gradient, c = vertical intercept



Exponential functions:

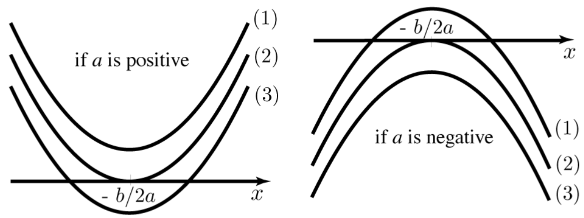
e ≈ 2.718 is the exponential constant.



Graph of $y = e^x$ showing exponential growth

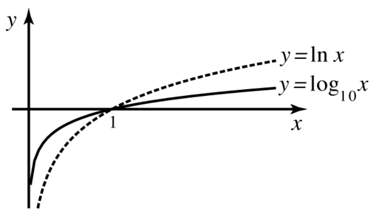
Graph of $y = e^{-x}$ showing exponential decay

Quadratic functions: $y = ax^2 + bx + c$



If a is positive	If a is negative
(1) $b^2 - 4ac < 0$	(1) $b^2 - 4ac > 0$
(2) $b^2 - 4ac = 0$	(2) $b^2 - 4ac = 0$
(3) $b^2 - 4ac > 0$	(3) $b^2 - 4ac < 0$

Logarithmic functions:



Graphs of $y = \ln x$ and $y = \log_{10} x$

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Arithmetic

When multiplying or dividing positive and negative numbers the sign of the result is given by:

positive×positive = positive positive×negative = negative
negative×positive = negative negative×negative = positive

$$\frac{\text{positive}}{\text{positive}} = \text{positive} \qquad \frac{\text{positive}}{\text{negative}} = \text{negative}$$

$$\frac{\text{negative}}{\text{positive}} = \text{negative} \qquad \frac{\text{negative}}{\text{negative}} = \text{positive}$$

The BODMAS rule reminds us of the order in which operations are carried out. BODMAS stands for:

Brackets ()	First priority
Of ×	Second priority
Division ÷	Second priority
Multiplication ×	Second priority
Addition +	Third priority
Subtraction −	Third priority

Fractions.

$$\text{fraction} = \frac{\text{numerator}}{\text{denominator}}$$

Adding and subtracting fractions. To add or subtract two fractions first rewrite each fraction so that they have the same denominator. Then, the numerators are added or subtracted as appropriate and the result is divided by the common denominator: e.g.

$$\frac{4}{5} + \frac{3}{4} = \frac{16}{20} + \frac{15}{20} = \frac{31}{20}$$

Multiplying fractions. To multiply two fractions, multiply their numerators and then multiply their denominators: e.g.

$$\frac{3}{7} \times \frac{5}{11} = \frac{15}{77}$$

Dividing fractions. To divide two fractions, invert the second and then multiply: e.g.

$$\frac{3}{5} \div \frac{2}{3} = \frac{3}{5} \times \frac{3}{2} = \frac{9}{10}$$

Algebra

Removing brackets:

$$a(b+c) = ab + ac, \qquad a(b-c) = ab - ac$$

$$(a+b)(c+d) = ac + ad + bc + bd$$

Formula for solving a quadratic equation:

$$\text{if } ax^2 + bx + c = 0 \text{ then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Laws of Indices:

$$a^m a^n = a^{m+n} \qquad \frac{a^m}{a^n} = a^{m-n} \qquad (a^m)^n = a^{mn}$$

$$a^0 = 1 \qquad a^{-m} = \frac{1}{a^m} \qquad a^{1/n} = \sqrt[n]{a} \qquad a^{\frac{m}{n}} = (\sqrt[n]{a})^m$$

Laws of Logarithms:

$y = \log_b x$ means $b^y = x$ and b is called the **base**.
e.g. $\log_{10} 2 = 0.3010$ means $10^{0.3010} = 2.000$, to 4 s.f.

$$\log_b AB = \log_b A + \log_b B, \qquad \log_b \frac{A}{B} = \log_b A - \log_b B,$$

$$\log_b A^n = n \log_b A, \qquad \log_b 1 = 0, \qquad \log_b b = 1$$

Logarithms to base e, denoted $\log_e$ or alternatively ln are called *natural logarithms*. The letter e stands for the exponential constant which is approximately 2.718.

Proportion and Percentage

To convert a fraction to a percentage multiply by 100 and label the result as a percentage.

Examples

$\frac{5}{8}$ as a percentage is $\frac{5}{8} \times 100\% = 62.5\%$
$\frac{1}{3}$ as a percentage is $\frac{1}{3} \times 100\% = 33\frac{1}{3}\%$

Some common conversions are

$$\frac{1}{10} = 10\%, \quad \frac{1}{4} = 25\%, \quad \frac{1}{2} = 50\%, \quad \frac{3}{4} = 75\%$$

Ratios are simply an alternative way of expressing fractions. Consider dividing £200 between two people in the ratio of 3:2. This means that for every £3 the first person gets, the second person gets £2. So the first gets $\frac{3}{5}$ of the total, and the second gets $\frac{2}{5}$ of the total; that is £120 and £80.

Generally, to split a quantity in the ratio $m:n$, the quantity is divided into $\frac{m}{m+n}$ of the total and $\frac{n}{m+n}$ of the total.

Sigma notation

The Greek capital letter sigma, Σ, is used as an abbreviation for an addition sum. Suppose we have n values $x_1, x_2, \ldots x_n$ and we wish to add them together. The sum

$$x_1 + x_2 + \ldots x_n \text{ is written } \sum_{i=1}^{n} x_i$$

Note that i runs through all whole number values from 1 to n. So, for instance

$$\sum_{i=1}^{3} x_i \text{ means } x_1 + x_2 + x_3$$

Example

$$\sum_{i=1}^{5} i^2 \text{ means } 1^2 + 2^2 + 3^2 + 4^2 + 5^2$$

Statistics

Population values, or **parameters**, are denoted by Greek letters. Population mean = μ. Population variance = σ^2. Population standard deviation = σ. Sample values, or **estimates**, are denoted by roman letters.

The **mean** of a sample of n observations $x_1, x_2, \ldots x_n$ is

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{x_1 + x_2 + \ldots + x_n}{n}$$

The unbiased estimate of the **variance** of these n sample observations is

$$s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1} \text{ which can be written as}$$

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n} x_i^2 - \frac{n\bar{x}^2}{n-1}$$

The sample unbiased estimate of **standard deviation**, s, is the square root of the variance:

$$s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

The Greek alphabet

A	α	alpha	I	ι	iota	P	ρ	rho
B	β	beta	K	κ	kappa	Σ	σ	sigma
Γ	γ	gamma	Λ	λ	lambda	T	τ	tau
Δ	δ	delta	M	μ	mu	Υ	υ	upsilon
E	ϵ	epsilon	N	ν	nu	Φ	ϕ	phi
Z	ζ	zeta	Ξ	ξ	xi	X	χ	chi
H	η	eta	O	o	omicron	Ψ	ψ	psi
Θ	θ	theta	Π	π	pi	Ω	ω	omega