

Division of complex numbers

sigma-complex7-2009-1

In this unit we are going to look at how to divide a complex number by another complex number.

Division of complex numbers relies on two important principles. The first is that multiplying a complex number by its conjugate produces a purely real number. The second principle is that both the numerator and denominator of a fraction can be multiplied by the same number, and the value of the fraction will remain unchanged.

For example, starting with the fraction $\frac{1}{2}$, we can multiply both top and bottom by 5 to give $\frac{5}{10}$, and the value of this is the same as $\frac{1}{2}$. We say that $\frac{1}{2}$ and $\frac{5}{10}$ are equivalent fractions.

Example. Suppose we want to divide the complex number $(4 + 7i)$ by $(1 - 3i)$, that is we want to find

$$\frac{4 + 7i}{1 - 3i}$$

We won't change the value of this fraction if we multiply both numerator and denominator by the same value. We multiply by the conjugate of the denominator, which is $1 + 3i$, and then simplify.

$$\begin{aligned} \frac{(4 + 7i)(1 + 3i)}{(1 - 3i)(1 + 3i)} &= \frac{4 + 12i + 7i + 21i^2}{1 + 3i - 3i - 9i^2} \\ &= \frac{-17 + 19i}{10} \\ &= -\frac{17}{10} + \frac{19}{10}i \\ &= -1.7 + 1.9i \end{aligned}$$

Example. Suppose we want to divide the complex number $(2 - 5i)$ by $(-4 + 3i)$, that is we want to find

$$\frac{2 - 5i}{-4 + 3i}$$

We multiply by the conjugate of the denominator, which is $-4 - 3i$, and then simplify.

$$\begin{aligned} \frac{(2 - 5i)(-4 - 3i)}{(-4 + 3i)(-4 - 3i)} &= \frac{-8 - 6i + 20i + 15i^2}{16 + 12i - 12i - 9i^2} \\ &= \frac{-23 + 14i}{25} \\ &= -\frac{23}{25} + \frac{14}{25}i \\ &= -0.92 + 0.56i \quad (2dp) \end{aligned}$$

In the next unit we will introduce the Argand Diagram, which is a graphical way of representing complex numbers.